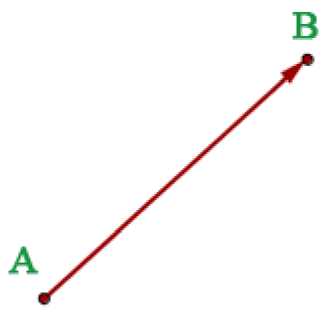
Unit 8. ANALYTIC GEOMETRY.

1. VECTORS IN THE PLANE

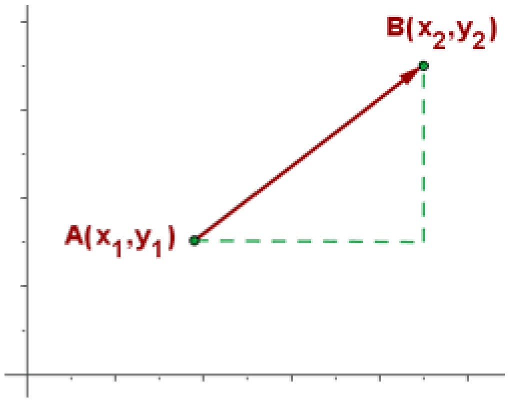


A **vector** is a line segment running from point A (tail) to point B (head).

1.1 DIRECTION OF A VECTOR

The direction of a vector is the direction of the line which contains the vector or any line which is parallel to it.

1.2 COMPONENTS OR COORDINATES OF A VECTOR



If the coordinates of A and B are: $A(x_1, y_1)$ $B(x_2, y_2)$, then the coordinates of the vector $\vec{AB}$ are: $\vec{AB} = (x_2 - x_1, y_2 - y_1)$

Solved Examples:

Example 1: Find the components of the vector $\overrightarrow{AB}$:

$A(2, 2) \qquad B(5, 7)$

$\overrightarrow{AB} = (5-2,\ 7-2) \qquad \overrightarrow{AB} = (3,\ 5)$

Example 2: The vector $\overrightarrow{AB}$ has the components (5, -2). Find the coordinates of A if the terminal point is known as B(12, -3).

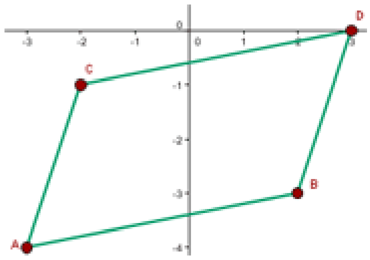
$$(12 - x_A, -3 - y_A) = (5, -2)$$

$$12 - x_A = 5 \qquad x_A = 7$$

$$A = (7, -1)$$

$$-3 - y_A = -2 \qquad y_A = -1$$

Example 3: Calculate the coordinates of Point D so that points A(-1, -2), B(4, -1), C(5, 2) and D form a parallelogram.



$$\overrightarrow{AB} = \overrightarrow{DC}$$

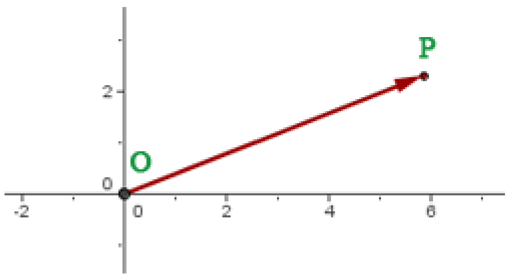
$$(4+1, -1+2) = (5 - x_D, 2 - y_D)$$

$$5 = 5 - x_D \qquad x_D = 0$$

$$D = (0, 1)$$

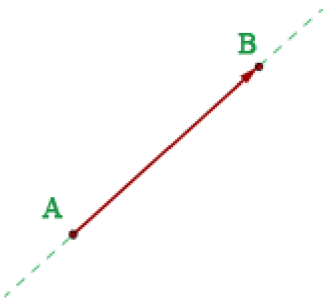
$$1 = 2 - y_D \qquad y_D = 1$$

1.3 POSITION VECTOR



The vector $\overrightarrow{OP}$ that joins the origin coordinates, **O=(0,0)**, with a point, **P**, is the position vector of point P.

1.4 MAGNITUDE OF A VECTOR



The magnitude of the vector $\overrightarrow{AB}$ is the length of the line segment $\overline{AB}$. It is denoted by $|\overrightarrow{AB}|$.

The magnitude of a vector is always a positive number or zero.

$$\vec{u} = (u_1, u_2)$$

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2}$$

The magnitude of a vector can be calculated (using the Pythagoras Theorem) if the coordinates of the endpoints are known:

$$A = (x_1, y_1) \qquad B = (x_2, y_2)$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solved examples:

Example 1: Calculate the magnitude of the following vector $\vec{u}$:

$$\vec{u} = (3, 4) \qquad |\vec{u}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Example 2: Calculate the magnitude of the vector determined by the endpoints A and B:

$$A = (2, 1) \quad B = (-3, 2) \qquad |\overrightarrow{AB}| = \sqrt{(-3-2)^2 + (2-1)^2} = \sqrt{26}$$

Example 3: Calculate the value of k knowing the magnitude of the vector $\vec{v}$ = (k, 3) is 5.

$$5 = \sqrt{k^2 + 9} \qquad 25 = k^2 + 9 \qquad k = \pm 4$$

1.5 UNIT VECTOR

The unit vector has a magnitude of one.

1.6 NORMALISING A VECTOR

Normalising a vector is obtaining a unit vector in the same direction.

To normalise a vector, divide the vector by its magnitude.

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

Examples:

If $\vec{v}$ is a vector of components (3, 4), find a unit vector in the same direction.

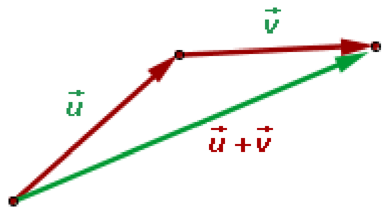
$$\vec{v} = (3, 4) \qquad |\vec{v}| = \sqrt{3^2 + 4^2} = 5 \quad \vec{u} = \frac{1}{5} \cdot (3, 4) \qquad \vec{u} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

2. OPERATION WITH VECTORS

2.1 ADDING VECTORS

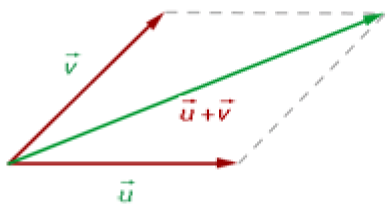
To add two vectors $\vec{u}$ and $\vec{v}$ graphically, join the tail of one with the head of the other vector.

The vector sum equals the distance from the tail of the first vector to the head of the second vector.



Parallelogram Rule

If there are two vectors with a common origin and parallel lines to the vectors are drawn, a parallelogram is obtained whose diagonal coincides with the sum of the vectors.



To add two vectors, add their components.

$$\vec{u}=(u_1, u_2) \qquad \vec{v}=(v_1, v_2)$$

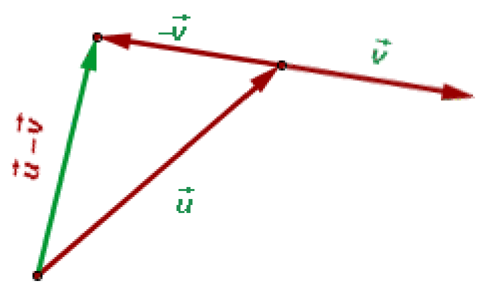
$$\vec{u}+\vec{v}=(u_1+v_1, \ u_2+v_2)$$

2.2 SUBTRACTING VECTORS.

To subtract two vectors $\vec{u}$ and $\vec{v}$, add $\vec{u}$ with the opposite of $\vec{v}$.

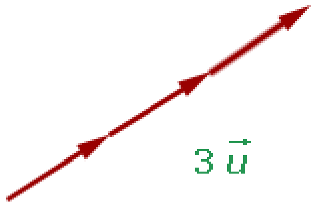
$$\vec{u}=(u_1, u_2) \qquad \vec{v}=(v_1, v_2)$$

$$\vec{u}-\vec{v}=(u_1-v_1, \ u_2-v_2)$$



2.3 SCALAR MULTIPLICATION

The product of a number, **k**, by a vector $\vec{u}$ is another vector. The result is another vector in the same direction of $\vec{u}$ if k is positive or in the opposite direction of $\vec{u}$ if k is negative. The magnitude of the vector is $|k| \cdot |\vec{u}|$.



$$\vec{u} = (u_1, u_2)$$

$$k \cdot (u_1, u_2) = (k \cdot u_1, k \cdot u_2)$$

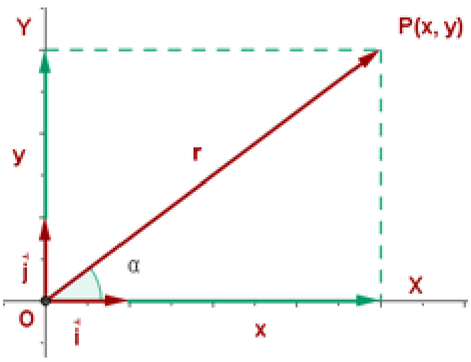
Example:

$$\vec{u} = (-2, 5) \qquad \vec{v} = (3, -1)$$

$$-\vec{u} = (2, -5)$$

$$3 \cdot \vec{v} = (9, -3)$$

3. CARTESIAN COORDINATES



In a system formed by a point, ***O***, and an orthonormal basis at each point, **P**, there is a corresponding vector, $\overrightarrow{OP}$, on the plane such that:

$$\overrightarrow{OP} = x\vec{i} + y\vec{j}$$

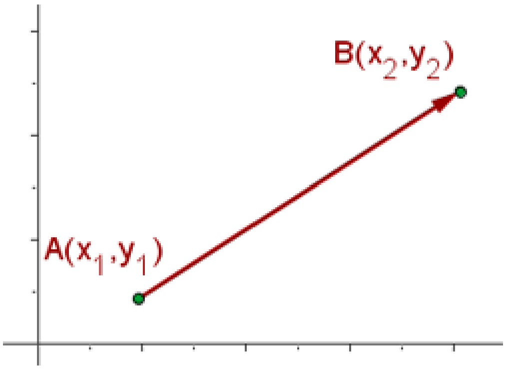
The coefficients x and y of the linear combination are called coordinates of point P.

The first, x, is the abscissa.

The second, y, is the ordinate.

As the linear combination is unique, each point corresponds to a pair of numbers and a each pair of numbers to a point.

4. DISTANCE BETWEEN TWO POINTS



$A(x_1, y_1)$ $B(x_2, y_2)$

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

Calculate the distance between points A(2, 1) and B(−3, 2).

$$d(AB) = \sqrt{(-3-2)^2 + (2-1)^2} = \sqrt{26}\ u$$

Solved Problems:

1. Prove that the points: A (1, 7), B (4,6) and C (1, -3) belong to the circumference of a circle whose centre is (1, 2).

If O is the centre of the circle, the distances from O to A, B, and C should be equal:

$$d(O, A) = \sqrt{(1-1)^2 + (7-2)^2} = 5$$

$$d(O, B) = \sqrt{(4-1)^2 + (6-2)^2} = 5$$

$$d(O, C) = \sqrt{(1-1)^2 + (-3-2)^2} = 5$$

2. Identify the type of triangle determined by points *A* (4, -3), B (3, 0) and *C* (0, 1).

$$d(A,B)=\sqrt{(3-4)^2+(0+3)^2}=\sqrt{10}$$

$$d(B,C)=\sqrt{(0-3)^2+(1-0)^2}=\sqrt{10}$$

$$d(A,C)=\sqrt{(0-4)^2+(1+3)^2}=\sqrt{32}=4\sqrt{2}$$

$$d(A,B)=d(B,C)\neq d(A,C)$$

It is an isosceles triangle.

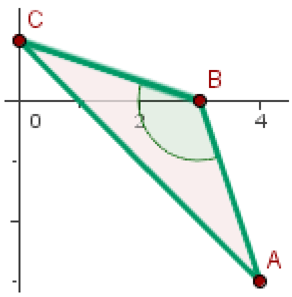
If:

$(d(A,C))^2<(d(A,B))^2+(d(B,C))^2$ Acute Triangle

$(d(A,C))^2=(d(A,B))^2+(d(B,C))^2$ Right Triangle

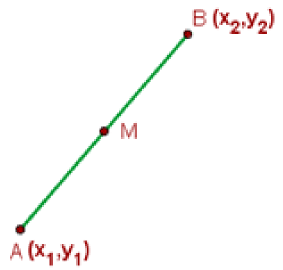
$(d(A,C))^2>(d(A,B))^2+(d(B,C))^2$ Obtuse Triangle

$32>10+10$ Obtuse Triangle



3. Calculate the value of a if the distance between points *A* = (0, a) and B = (1, 2) is equal to 1.

5. MIDPOINT



$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Solved examples:

1. Calculate the coordinates of the midpoint of line segment AB.

$$A = (3,\ 9) \qquad B = (-1,\ 5)$$

$$x_M = \frac{3-1}{2} \qquad y_M = \frac{9+5}{2}$$

$$M = (1,\ 7)$$

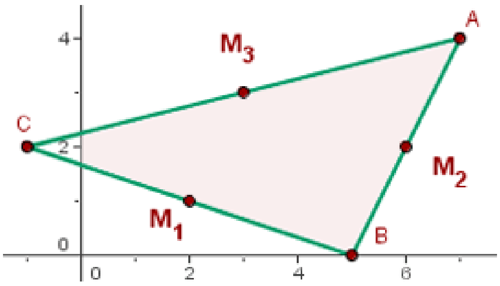
2. Calculate the coordinates of Point C in the line segment AC, knowing that the midpoint is B = (2, -2) and an endpoint is A = (-3, 1).

$$2 = \frac{-3 + x_c}{2} \qquad 4 = -3 + x_c \qquad x_c = 7$$

$$C = (7,\ -5)$$

$$-2 = \frac{1 + y_c}{2} \qquad -4 = 1 + y_c \qquad y_c = -5$$

3. If M_1= (2, 1), M_2= (3, 3) and M_3= (6, 2) are the midpoints of the sides that make up a triangle, what are the coordinates of the vertices?



$$\begin{cases} 6 = \dfrac{x_1 + x_2}{2} \\ 2 = \dfrac{x_2 + x_3}{2} \\ 3 = \dfrac{x_3 + x_1}{2} \end{cases} \qquad \begin{cases} 2 = \dfrac{y_1 + y_2}{2} \\ 1 = \dfrac{y_2 + y_3}{2} \\ 3 = \dfrac{y_3 + y_1}{2} \end{cases}$$

$x_1 = 7$; $x_2 = 7$; $x_3 = -1$

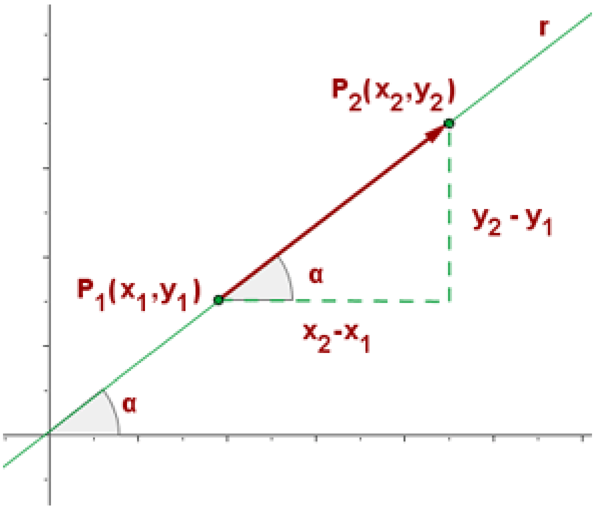
$y_1 = 4$; $y_2 = 0$; $y_3 = 3$

A=(7, 4);B=(5, 0) C=(-1, 2)

6. SLOPE

The slope is the inclination of a line with respect to the x-axis.

It is denoted by the letter **m**.



Slope given two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope given the angle:

$$m = tg\ \alpha$$

Slope given a vector of the line:

$$m = \frac{v_2}{v_1}$$

Slope given the equation of the line:

$$m = -\frac{A}{B}$$

Two lines are parallel if their slopes are equal.

$$m_r = m_s$$

Two lines are perpendicular if their slopes are the inverse of each other and their signs are opposite.

$$m_s = -\frac{1}{m_r}$$

Solved Examples:

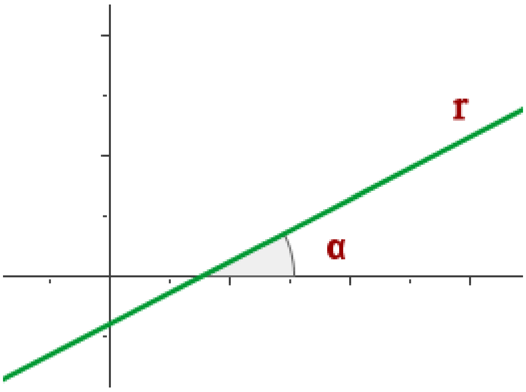
Ex 1: The slope of the line that passes through points A = (2, 1) and B = (4, 7) is:

$$m = \frac{7-1}{4-2} = 3$$

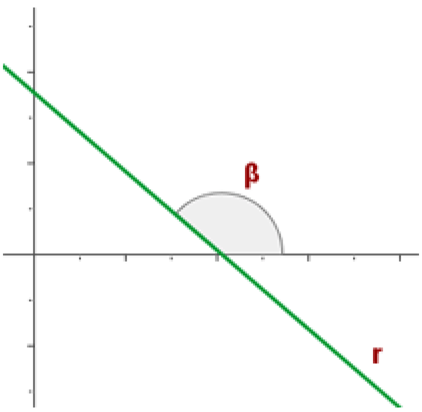
Ex 2: The line passes through points A = (1, 2) and B = (1, 7) and has no slope, since division by 0 is undefined.

$$m = \frac{7-2}{1-1} = \frac{5}{0}$$

If the angle between the line and the positive x-axis is acute, the slope is positive and grows as the angle increases.



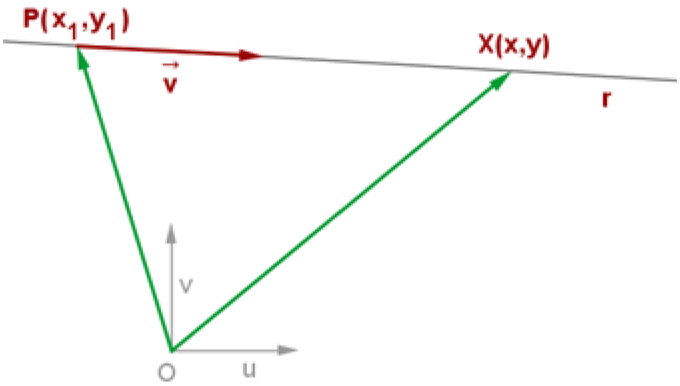
If the angle between the line and the positive x-axis is obtuse, the slope is negative and diminishes as the angle increases.



7. DIFFERENT FORMULAE TO DEFINE A LINE

7.1 VECTOR EQUATION OF A LINE

A line is defined as the set of aligned points on the plane with a point, P, and a directional vector $\vec{v}$.



If $P(x_1, y_1)$ is a point on the line and the vector $\overrightarrow{PX}$ has the same direction as $\vec{v}$, then $\overrightarrow{PX}$ equals $\vec{v}$ multiplied by a scalar unit:

$$\overrightarrow{PX} = t \cdot \vec{v} \qquad t \in \mathbb{R}$$

$$\overrightarrow{PX} = \overrightarrow{OX} - \overrightarrow{OP} \qquad \overrightarrow{OX} - \overrightarrow{OP} = t \cdot \vec{v}$$

$$\overrightarrow{OX} = \overrightarrow{OP} + t \cdot \vec{v}$$

$$(x, y) = (x_1, y_1) + t \cdot (v_1, v_2)$$

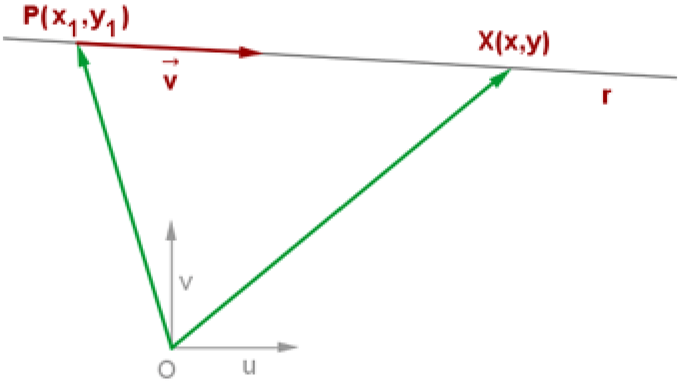
Solved Examples:

Ex 1. A line passes through point A = (-1, 3) and has a directional vector $\vec{v}$ with components (2, 5). Determine the equation of the vector.

$$(x, y) = (-1, 3) + k \cdot (2, 5)$$

Ex 2. Write the vector equation of the line which passes through points A = (1, 2) and B = (-2, 5).

7.2 PARAMETRIC FORM



$$\overrightarrow{OX} = \overrightarrow{OP} + t \cdot \vec{v}$$

$$(x, y) = (x_1, y_1) + t \cdot (v_1, v_2)$$

$$(x, y) = (x_1 + t \cdot v_1, y_1 + t \cdot v_2)$$

$$\begin{cases} x = x_1 + t \cdot v_1 \\ y = y_1 + t \cdot v_2 \end{cases}$$

Solved Examples:

Ex 1. A line through point A =(−1, 3) has a direction vector of $\vec{v}$ = (2, 5). Write the equation for this vector in parametric form.

$$\begin{cases} x = -1 + 2k \\ y = 3 + 5k \end{cases}$$

Ex 2. Write the equation of the line which passes through points A = (1, 2) and B = (−2, 5) in parametric form.

7.3 POINT-SLOPE FORM

$$y - y_1 = m(x - x_1)$$

m is the **slope** of the line, and (x_1, y_1) is any point on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = tg\ \alpha$$

$$m = \frac{v_2}{v_1}$$

Solved Examples:

Ex 1. Calculate the point-slope form equation of the line passing through points A = (-2, -3) and B = (4, 2).

$$m = \frac{2+3}{4+2} = \frac{5}{6}$$

$$y + 3 = \frac{5}{6}(x + 2)$$

Ex 2. Calculate the point-slope form equation of the line with a slope of 45° which passes through point (-2, -3).

$$tg\ 45° = 1$$

$$y + 3 = x + 2$$

Ex 3. A line passes through Point A(-1, 3) and has a direction vector of $\vec{v}$ = (2, 5). Write the equation of the line in point-slope form.

$$y - 3 = \frac{5}{2}(x + 1)$$

7.4 TWO-POINTS FORM

If the values of parameter t in the parametric equations are equal, then:

$$\begin{cases} x = x_1 + t \cdot v_1 \\ y = y_1 + t \cdot v_2 \end{cases}$$

$$t = \frac{x - x_1}{v_1}$$

$$t = \frac{y - y_1}{v_2}$$

$$\frac{x - x_1}{v_1} = \frac{y - y_1}{v_2}$$

$$\frac{x - x_1}{v_1} = \frac{y - y_1}{v_2}$$

$v_1 = x_2 - x_1 \, v_2 = y_2 - y_1$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

The two-point form equation of the line can also be written as:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Solved Example:

Ex 1: Determine the equation of the line that passes through points A = (1, 2) and B = (−2, 5).

$$\frac{x-1}{-2-1} = \frac{y-2}{5-2}$$

7.5 GENERAL FORM

$$Ax + By + C = 0$$

A, B and C are constants, and the values of A and B cannot both be equal to zero.

The equation is usually written with a positive value for A.

The slope of the line is:

$$m = -\frac{A}{B}$$

The director vector is:

$$\vec{v} = (-B,\ A)$$

Solved Examples:

Ex 1: Determine the equation in general form of the line that passes through point A = (1, 5) and has a slope of m = −2.

$y - 5 = -2(x - 1) \qquad y - 5 = -2x + 2$

$$2x + y - 7 = 0$$

Ex 2: Write the equation in general form of the line that passes through points A = (1, 2) and B = (−2, 5).

$$\frac{x-1}{-2-1} = \frac{y-2}{5-2}$$

$$x + y - 3 = 0$$

7.6 SLOPE-INTERCEPT FORM

If the value of **y** in the general form equation is isolated, the slope-intercept form of the line is obtained:

$$y = -\frac{A}{B}x - \frac{C}{B}$$

$$y = mx + b$$

The coefficient of x is the slope, which is denoted as **m**.

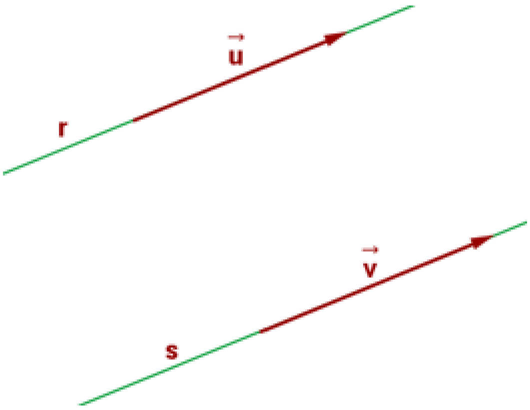
The independent term is the y-intercept, which is denoted as **b**.

Example: Calculate the equation (in slope-intercept form) of the line that passes through point A = (1,5) and has a slope m = -2.

$$y - 5 = -2(x-1) \qquad y - 5 = -2x + 2$$

$$y = -2x + 7$$

8. PARALLEL LINES



Two lines are parallel if their slopes are equal.

$$m_r = m_s$$

Two lines are parallel if the respective coefficients of x and y are proportional.

$$\frac{A_1}{B_1} = \frac{A_2}{B_2}$$

Two lines are parallel if their directional vectors are equal.

$$\vec{v}_r = \vec{v}_s$$

Two lines are parallel if the angle between them is 0°.

$r \parallel s$ $\quad r \equiv Ax + By + C = 0 \qquad s \equiv Ax + By + k = 0 \quad k \in \mathbb{R}$

$\vec{v}_r = \vec{v}_s = (-B, A)$

$m_r = m_s = -\frac{A}{B} = \frac{v_2}{v_1}$

Solved Examples:

Ex 1: Calculate k so that the lines r ≡ x+2y-3 = 0 and s ≡ x−ky+4 = 0, are parallel.

$m_r = -\frac{1}{2} \qquad m_s = \frac{1}{k}$

$r \parallel s \qquad -\frac{1}{2} = \frac{1}{k} \qquad k = -2$

Ex 2: Determine the equation for the line parallel to r ≡ x+2y+3 = 0 that passes through point A = (3, 5).

$m_r \parallel m_s$

$m_r = m_s = -\frac{1}{2}$

$y - 5 = -\frac{1}{2}(x - 3) \qquad 2y - 10 = -x + 3$

$x + 2y - 13 = 0$

Ex 3: Determine the equation for the line parallel to r ≡ 3x+2y−4 = 0 that passes through point A = (2, 3).

$3 \cdot 2 + 2 \cdot 3 + k = 0 \qquad k = -12$

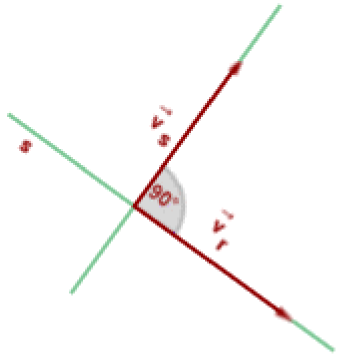
3x + 2y - 12= 0

Ex 4: The line r ≡ 3x+ny−7 = 0 passes through point A = (3, 2), and it is parallel to the line s ≡ mx+2y−13 = 0. Calculate the values of m and n.

$A \in r \qquad 3 \cdot 3 + n \cdot 2 - 7 = 0 \qquad n = -1$

$r \parallel s \qquad \frac{3}{m} = \frac{-1}{2} \qquad m = -6$

9. PERPENDICULAR LINES



If two lines are perpendicular, their **slopes** are the inverse of each other and their signs are opposite.

$$m_s = -\frac{1}{m_r}$$

Two lines are perpendicular if their directional vectors are perpendicular.

$$r \perp s \qquad \begin{array}{lll} r \equiv Ax + By + C = 0 & & \vec{v}_r = (-B, A) \\ s \equiv -Bx + Ay + k = 0 & k \in \mathbb{R} & \vec{v}_s = (A, B) \end{array}$$

Solved Examples:

Ex 1: Determine the equation of the line that is perpendicular to r≡x+2y+3=0 and passes through point A=(3, 5).

$$m_r \perp m_s$$

$$m_r = -\frac{1}{2} \qquad m_s = 2$$

$$y - 5 = 2 \cdot (x - 3)$$

$$2x - y - 1 = 0$$

Ex 2: Given the lines r ≡ 3x + 5y − 13 = 0 and s ≡ 4x − 3y + 2 = 0, calculate the equation of the line that passes through their point of intersection and is perpendicular to line t ≡ 5x − 8y + 12 = 0.

$$\begin{cases} 3x + 5y - 13 = 0 \\ 4x - 3y + 2 = 0 \end{cases} \qquad P = (1, 2) \qquad m = \frac{8}{5}$$

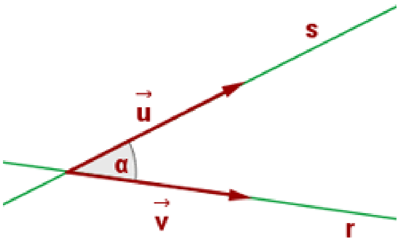
$$y - 2 = -\frac{8}{5}(x - 1) \qquad 8x + 5y - 18 = 0$$

Ex 3: Calculate **k** so that lines r ≡ x + 2y − 3 = 0 and s ≡ x − ky + 4 = 0 are perpendicular.

$m_r = -\frac{1}{2}$ $\qquad m_s = \frac{1}{k}$

$r \perp s \qquad -\frac{1}{2} = \frac{1}{-\frac{1}{k}} \qquad \frac{1}{2} = k$

10. ANGLE BETWEEN TWO LINES



The angle between two lines is the smaller of the angles formed by the intersection of the two lines. The angle can be obtained from:

1. Their slopes:

$$tg\ \alpha = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|$$

2. Their director vectors:

$$\cos \alpha = \frac{|u_1 \cdot v_1 + u_2 . v_2|}{\sqrt{u_1^2 + u_2^2} \cdot \sqrt{v_1^2 + v_2^2}}$$

Solved Examples:

Ex 1: Find the angle between lines r and s, if their directional vectors are: $\vec{u} = (-2, 1)$ and $\vec{v} = (2, -3)$.

$$\cos\ \alpha = \frac{|(-2,1)\cdot(2,-3)|}{\sqrt{5}\cdot\sqrt{13}} = \frac{|-7|}{\sqrt{65}} = 0.868$$

$$\alpha = 29^\circ\ 44'$$

Ex 2: Given the lines r ≡ 3x + y - 1 = 0 and s ≡ 2x + my - 8 = 0, calculate the value of m so that both lines form an angle of 45°.

$\alpha = 45^\circ \qquad \cos 45^\circ = \frac{\sqrt{2}}{2}$

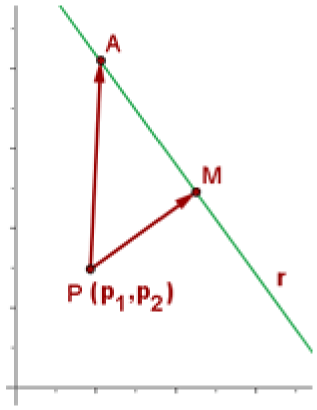
$\vec{v_r} = (-1,\ 3) \qquad \vec{v_s} = (-m,\ 2)$

$$\frac{\sqrt{2}}{2} = \frac{-1\cdot(-m)+3\cdot 2}{\sqrt{1+9}\cdot\sqrt{m^2+4}} \qquad \left(\frac{\sqrt{2}}{2}\right)^2 = \left(\frac{m+6}{\sqrt{1+9}\cdot\sqrt{m^2+4}}\right)^2$$

$$\frac{36+12m+m^2}{10(4+m^2)} = \frac{1}{2} \qquad m^2-2m-4=0$$

$$m_1 = 4 \qquad m_2 = -1$$

11. DISTANCE FROM A POINT TO A LINE



The distance from a point to a straight line is the length of a line segment drawn from the point that forms a perpendicular angle with the straight line.

$$d(P,r) = |\overrightarrow{PM}|$$

$$d(P,r) = \frac{|A\cdot p_1 + B\cdot p_2 + C|}{\sqrt{A^2+B^2}}$$

Solved Examples:

Ex 1: Find the distance from Point P = (2, −1) to the line r ≡ 3 x + 4 y = 0.

$$d(P,r) = \frac{|3\cdot 2+4\cdot(-1)|}{\sqrt{3^2+4^2}} = \frac{2}{5}$$

Ex 2: Calculate the distance between the following lines:

$$r \equiv x+3y-5=0 \qquad s \equiv x+3y+18=0$$

$$d(r,s) = \frac{|18+5|}{\sqrt{1^2+3^2}} = \frac{23}{\sqrt{10}}$$

EXERCISES

1. Write the equation (in all possible forms) of the line that passes through points A = (1, 2) and B = (2, 5).

2. Identify the type of triangle formed by these points: A = (6, 0), B = (3, 0) and C = (6, 3).

3. Determine the slope and y-intercept of this line: $3x + 2y - 7 = 0$.

4. Find the equation of the line r which passes through point A = (1, 5) and is parallel to line $s \equiv 2x + y + 2 = 0$.

5. Find the equation of the line that passes through point (2, −3) and is parallel to the straight line that joins points (4, 1) and (−2, 2).

6. Points A = (−1, 3) and B = (3, −3) are vertices of an isosceles triangle ABC that has its apex C on line $2x - 4y + 3 = 0$. If AC and BC are the equal sides, calculate the coordinates of point C.

7. Line $r \equiv 3x + ny - 7 = 0$ passes through point A = (3, 2) and is parallel to line $s \equiv mx + 2y - 13 = 0$. Calculate the values of m and n.

8. Given a triangle ABC with coordinates A = (0, 0), B = (4, 0) and C = (4, 4), calculate the equation of the angle bisector that passes through the vertex, C.

9. A parallelogram has a vertex A = (8, 0), and the point of intersection of its two diagonals is M = (6, 2). If the other vertex is at the origin, calculate:

   a) The other two vertices.

   b) The equations of the diagonals.

   c) The length of the diagonal.

10. Identify the type of triangle formed by points: A = (4, −3), B = (3, 0) and C = (0, 1).

11. Calculate the equation of the line that passes through point P = (−3, 2) and is perpendicular to line $r \equiv 8x - y - 1 = 0$.

12. Line $r \equiv x + 2y - 9 = 0$ is the perpendicular bisector of the line segment AB whose endpoint A has the coordinates (2, 1). Find the coordinates of the other endpoint.

13. Calculate the angle between the lines whose equations are:

$r_1 \equiv 3x + 4y - 12 = 0$ $\qquad$ $r_2 \equiv 6x + 8y + 1 = 0$

$s_1 \equiv 2x + 3y - 5 = 0$ $\qquad$ $s_2 \equiv 3x - 2y + 10 = 0$

14. A straight line is parallel to line $r \equiv 5x + 8y - 12 = 0$, and it is 6 units from the origin. What is the equation of this line?

15. Determine the equations of the angle bisectors formed by these lines:

$r \equiv 24x - 7y - 2 = 0$; $\qquad s \equiv 3x + 4y - 4 = 0$

16. The vertices of a parallelogram are $A = (3, 0)$, $B = (1, 4)$, $C = (-3, 2)$ and $D = (-1, -2)$. Calculate its area.

17. Given the triangle formed by points $A = (-1, -1)$, $B = (7, 5)$ and $C = (2, 7)$, calculate the equations of its heights and determine the orthocenter of the triangle.

18. A line is perpendicular to line $r \equiv 5x - 7y + 12 = 0$, and it is 4 units away from the origin. Determine the equation of this line.