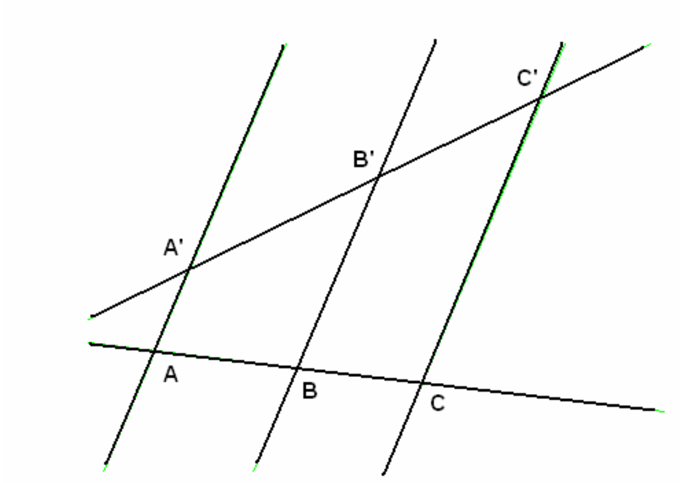
Unit 7. SIMILARITY AND TRIGONOMETRY.

1. THALES' THEOREM

1.1. THALES' THEOREM (INTERCEPT THEOREM)

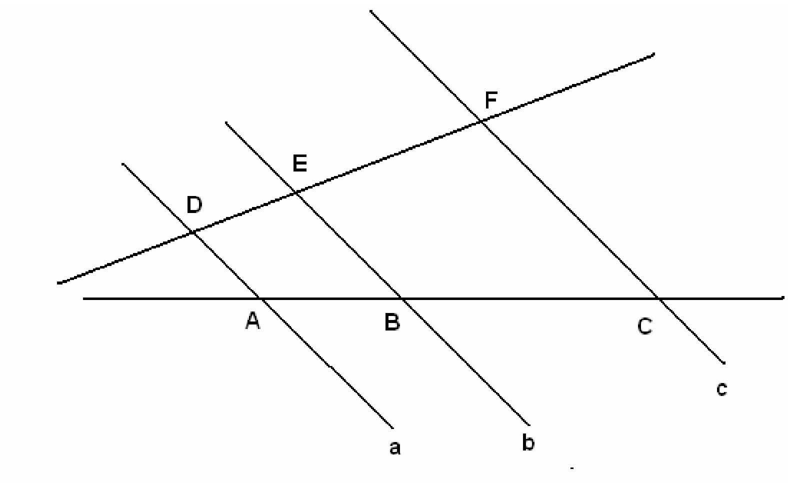
If two non parallel lines intersect with parallel lines, the ratios of any two segments on the first line are equal to the ratios of the corresponding segments on the second line.



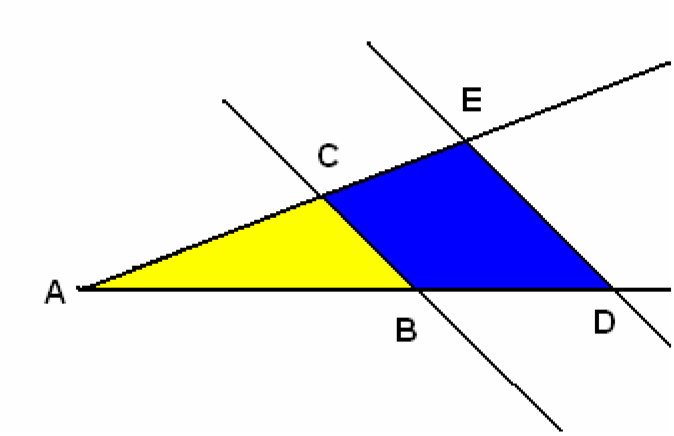
That is:

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$$

Example: Consider that lines a, b and c are parallel lines and AB = 2 cm, EF = 2.8 cm and BC = 3.5 cm. Calculate DE.

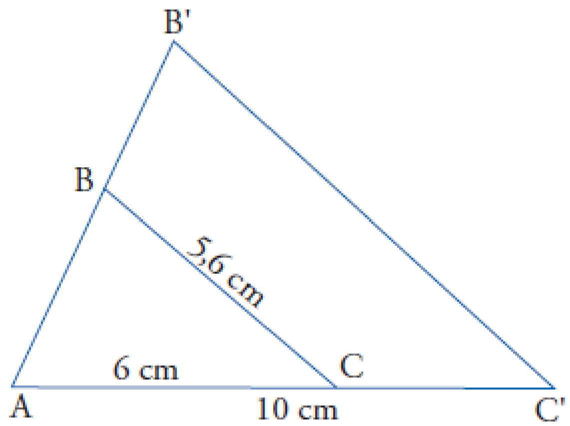


1.2. TRIANGLES IN THE THALES POSITION



If we use the Thales theorem in the drawing above in which the non parallel lines intercept at A, the triangle ABC is inside and fitted on the triangle ADE, we say that these two triangles are in the Thales position and they are similar because corresponding angles are equal and corresponding sides are in proportion.

Example: Use the Thales theorem to find B'C' in the picture below:

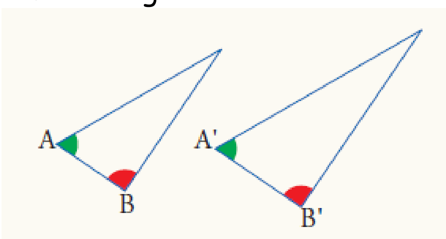


1.3. SIMILAR TRIANGLES. SIMILARITY CRITERIA

To be sure that two triangles are similar, we do not need to check that all three corresponding angles are equal and corresponding sides are in proportion. We can make sure that they are similar with fewer conditions. These sets of conditions are called **similarity criteria**.

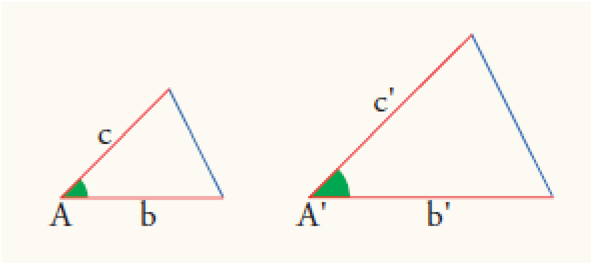
A) First similarity criterion

Two triangles ABC and A'B'C' are similar if $\hat{A} = \hat{A}'$ and $\hat{B} = \hat{B}'$.



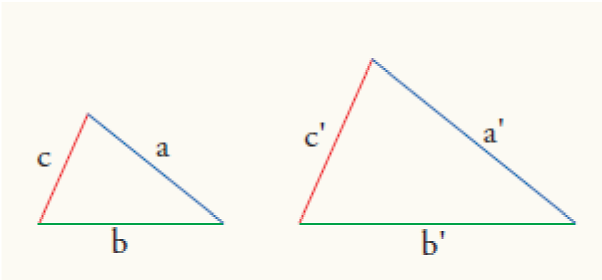
B) Second similarity criterion

Two triangles ABC and A'B'C' are similar if $\hat{A} = \hat{A}'$ and $\frac{b}{b'} = \frac{c}{c'}$



C) Third similarity criterion

Two triangles ABC and A'B'C' are similar if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$



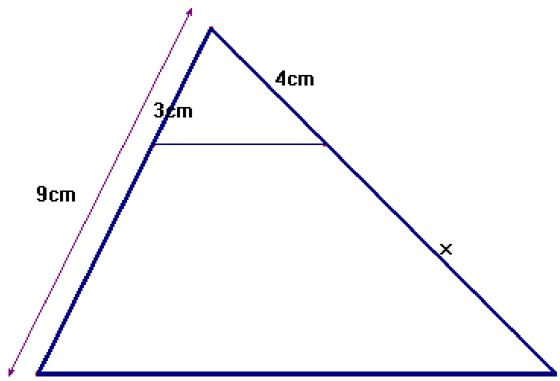
EXERCISE 1:

1.1 A flag pole casts a shadow 3 metres long. A woman near the pole casts a shadow 0.75 metres long. She is 1.5 metres tall. How tall is the flag pole?

1.2 Most TV screens have similar shapes. The measure of the diagonal is used to give screen size. Suppose the dimensions of a 9-inch screen are 5 inches by 7.5 inches. Find the dimensions of an 18-inch TV and a 36-inch TV.

1.3 The legs of a right triangle measure 12 m and 5 m. What is the length of the legs of a similar triangle to this one whose hypotenuse is 52 m?

1.4 In the triangle below, work out the length of x:

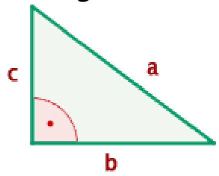


1.5

1.6 Andy wants to find the height of the tallest building in the city. He lies down on the ground 425 m from the building, and looks at a tree which is 38 m in front of him, which he knows is 21 m tall, and which looks as tall as the building from his position. How tall is the building in metres?

2. PITAGORAS' THEOREM

In a right triangle, the square of the hypotenuse equals the sum of the squares of the legs.



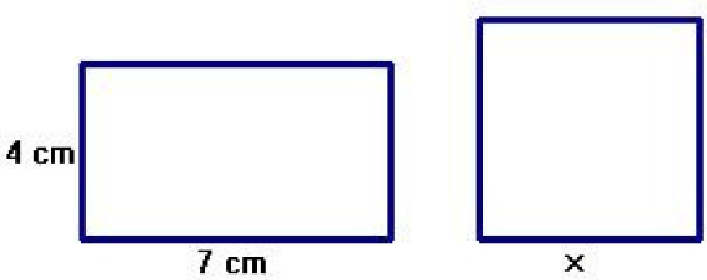
$$a^2 = b^2 + c^2$$

EXERCISE 2:

2.1 An isosceles triangle has sides which are 10 cm, 10 cm and 4 cm long. Find the height of the triangle.

2.2 Calculate the area of a regular hexagon whose side is 12 cm.

2.3 The square and the rectangle below have diagonals which are the same length. Find x.



2.4 A ship sails 20 km due north and then 35 km due east. How far it is from its starting point?

2.5 Find the length of the diagonal of a cube whose side is 3 dm.

2.6 Find the height of a rectangular box whose length is 8 cm and whose width is 6 cm, where the length of a diagonal is 11 cm. Calculate its volume.

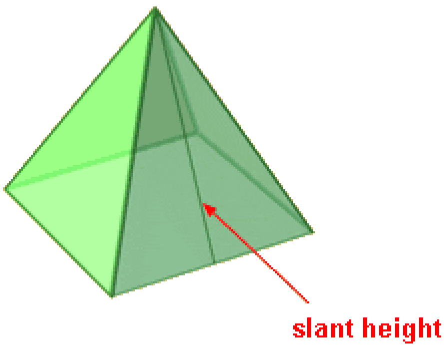
2.7 An equilateral triangle has a base of 7m. What are the height, the area and the perimeter of the triangle?

2.8 The perimeter of an isosceles trapezoid is 110 m, its bases are 40 and 30 m respectively. Calculate the length of its non-parallel sides and its area.

2.9 In a circle a chord 48 cm long is 7 cm from the centre. Calculate the area of the circle.

2.10 Calculate the lateral area, surface area and volume of a square pyramid whose base side is 10 cm long and whose height is 12 cm.

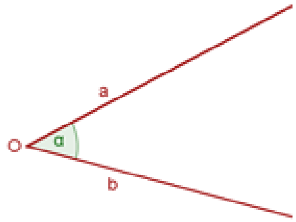
2.11 Calculate the lateral area, surface area and volume of a cone whose slant height[1] is 13 cm and whose base radius is 5 cm.



[1] The *slant height* of a cone is the length of the line segment from the apex perpendicular to an edge of the base:

3. MEASUREMENT OF ANGLES

An **angle** is the region of the plane between two rays with a common origin. The rays are called **sides** and have a common origin, the **vertex**.



An angle is positive if rotated in a counterclockwise direction and negative when rotated clockwise.

Angles are measured using the following units:

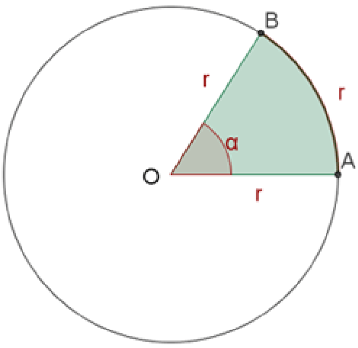
1. Sexagesimal degree (°)

If the circumference of a circle is divided into 360 equal parts, the central angle corresponding to each of its 360 parts is an angle of one sexagesimal degree (1°).

A **degree** has **60 minutes** (') and a **minute** has **60 seconds** ('').

2. Radian (rad)

It is the measure of an **angle** whose **arc** measures the same as the **radius**.



2π rad = 360°

π rad = 180°

How many radians is a 30° angle?

$$\frac{\pi}{\alpha}=\frac{180^\circ}{30^\circ} \qquad \alpha=\frac{30^\circ\pi}{180^\circ}=\frac{\pi}{6}rad$$

How many degrees is a π/3 rad angle?

$$\frac{\pi}{\frac{\pi}{3}}=\frac{180^\circ}{\alpha} \qquad \alpha=\frac{180^\circ\cdot\frac{\pi}{3}}{\pi}=\frac{180^\circ}{3}=60^\circ$$

EXERCISE 3:

3.1 Express the following angles in degrees:

a) 3 rad.

b) $2\pi/5$ rad.

c) $3\pi/10$ rad.

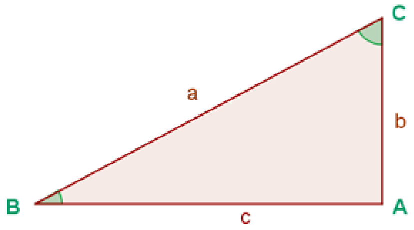
3.2 Express the following angles in radians:

a) 90°

b) 210°

c) 270°

4. TRIGONOMETRIC RATIOS



In a right triangle, the following trigonometric ratios can be defined:

1. Sine

The **sine** of angle B is the **ratio** between the length of the **opposite side** and that of the **hypotenuse** of the triangle. It is denoted by **sin B**.

$$\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{b}{a}$$

2. Cosine

The **cosine** of angle B is the **ratio** between the length of the **adjacent side** and that of the **hypotenuse** of the triangle. It is denoted by **cos B**.

$$\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{c}{a}$$

3. Tangent

The **tangent** of angle B is the **ratio** between the length of the **opposite side** and that of the **adjacent side** of the triangle. It is denoted by **tg B or tan B**.

$$\tan B = \frac{\sin B}{\cos B} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{c}$$

4. Cosecant

The **cosecant** of angle B is the **inverse** of the **sine** of B. It is denoted by **csc B**.

$$\csc B = \frac{1}{\sin B} = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{a}{b}$$

5. Secant

The **secant** of angle B is the **inverse** of the **cosine** of B. It is denoted by **sec B**.

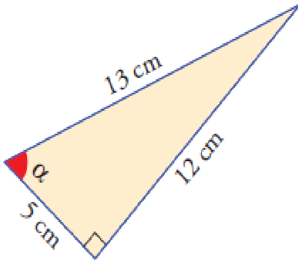
$$\sec B = \frac{1}{\cos B} = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{a}{c}$$

6. Cotangent

The **cotangent** of the angle B is the **inverse** of the **tangent** of B. It is denoted by **cot B**.

$$\cot B = \frac{1}{\tan B} = \frac{\cos B}{\sin B} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{c}{b}$$

Example: Calculate the trigonometric ratios of the angle α.



sen α = 12/13   cos α = 5/13   tg α = 12/5

cosec α = 13/12   sec α = 13/5   cotg α = 5/12

PROPERTIES:

A) $\sin^2\alpha + \cos^2\alpha = 1$

B) $tg\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

C) $-1 \le \sin\alpha \le 1$; $-1 \le \cos\alpha \le 1$

⚠ $(\sin\alpha)^2 = \sin^2\alpha$; $(\cos\alpha)^2 = \cos^2\alpha$; $(tg\,\alpha)^2 = tg^2\alpha$

Don't forget that sin α, cos α and tg α are real numbers.

Example: sin α = 3/5. Calculate cos α.

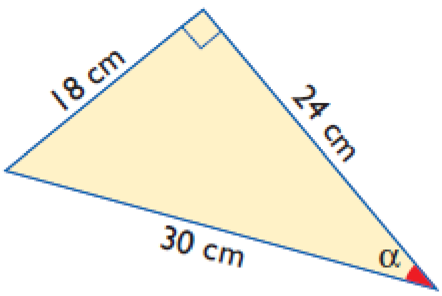
$$\text{sen}^2\alpha + \cos^2\alpha = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2\alpha = 1 \Rightarrow \frac{9}{25} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow$$

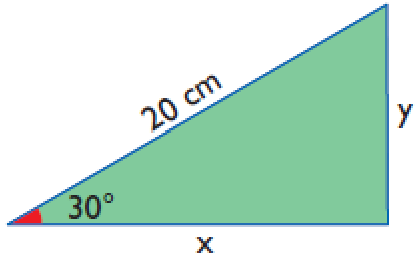
$$\cos\alpha = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

EXERCISE 4:

4.1 Calculate the trigonometric ratios of angle α:



4.2 Calculate the length of each leg in this right-angled triangle knowing that sin 30°=0.5 and cos 30°=0.8660.

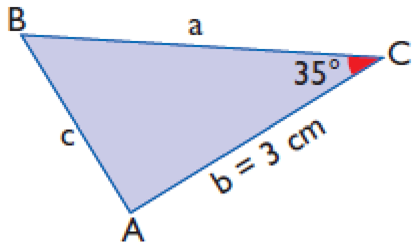


4.3 Calculate the angle α

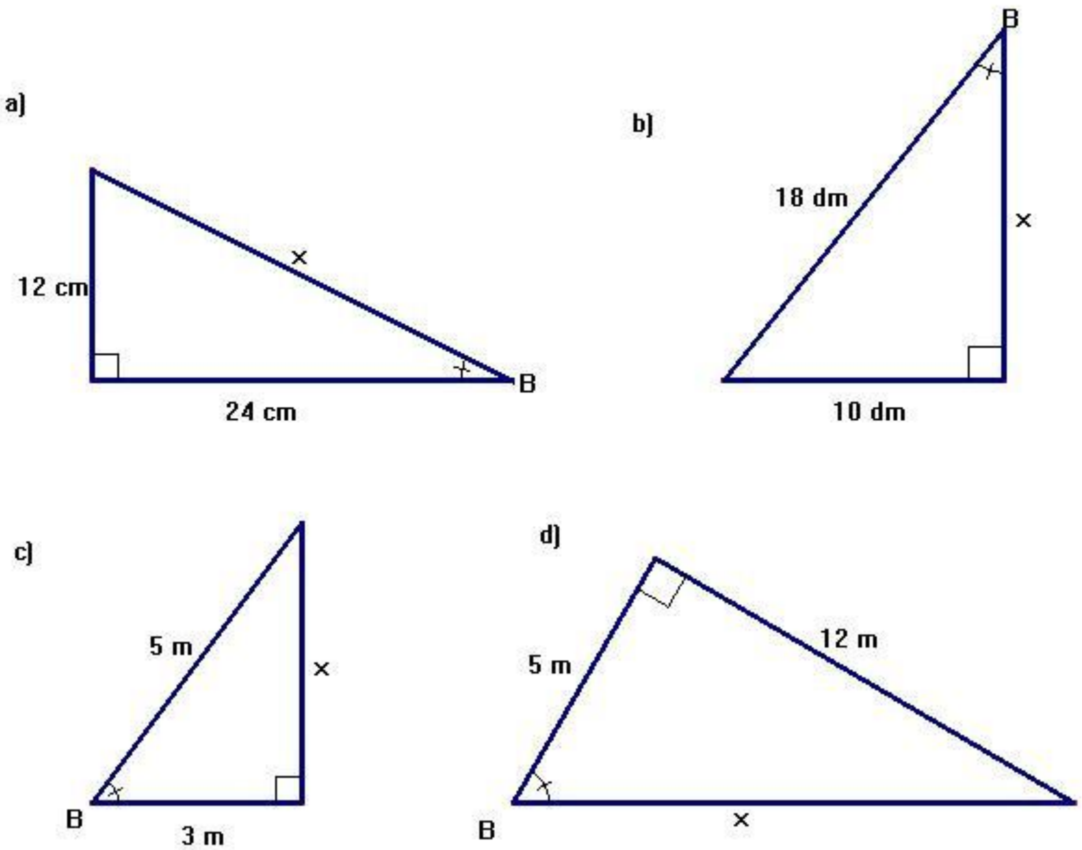
a) sen $\alpha = 0{,}8530$ b) cos $\alpha = 0{,}4873$

c) tg $\alpha = 0{,}7223$ d) cos $\alpha = 0{,}7970$

4.4 Work out a, c and B in this triangle:



4.5 Calculate the size of the angle marked in each triangle (B), in degrees and minutes. Also, calculate the length of the unknown side of each triangle (x).



4.6 Suppose cosα=2/5. Find out the other trigonometric ratios for α.

4.7 Suppose tgα=4. Find out the other trigonometric ratios for α.

4.8 Suppose sinα=1/4. Find out the other trigonometric ratios for α.

4.9 Suppose tgα= $\sqrt{3}$. Find out the other trigonometric ratios for α.

4.10 Simplify the following expressions:

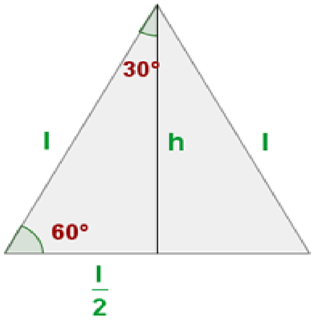
a) $\cos\alpha + \sin\alpha \cdot tg\alpha$

b) $\cos^3\alpha + \cos\alpha \cdot \sin^2\alpha$

5. TRIGONOMETRIC VALUES

Sine, Cosine and Tangent of 30° and 60°

In an **equilateral triangle**, all the angles measure **60°**. The **height**, **h**, divides the equilateral triangle into two equal right triangles. Using the **Pythagorean Theorem**, the **height** is:



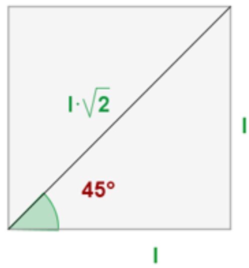
$$h = \sqrt{l^2 - \left(\frac{l}{2}\right)^2} = \sqrt{\frac{3l^2}{4}} = \frac{\sqrt{3}}{2}\, l$$

$$\sin 30^\circ = \frac{\frac{l}{2}}{l} = \frac{1}{2} \qquad \sin 60^\circ = \frac{\frac{\sqrt{3}}{2}\, l}{l} = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\frac{\sqrt{3}}{2}\, l}{l} = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{\frac{l}{2}}{l} = \frac{1}{2}$$

$$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60^\circ = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Sine, Cosine and Tangent of 45°



$$d = \sqrt{l^2+l^2} = \sqrt{2l^2} = l\sqrt{2}$$

$$\sin 45° = \frac{l}{l\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos 45° = \frac{l}{l\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Table of Trigonometric Values

α:	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\to \infty$

6. SOLVING RIGHT TRIANGLES

To solve a triangle, we must find the value of all the sides and angles.

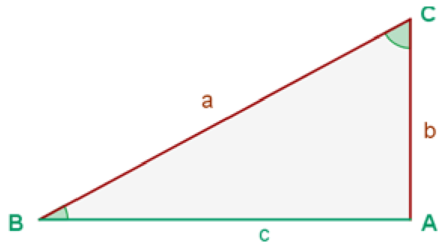
To solve a right triangle, we use **trigonometric ratios** and the **Pythagorean theorem**.

1st CASE: the hypotenuse and a leg are known:

$$B: \ \sin B = \frac{b}{a} \qquad B = \arcsin\frac{b}{a}$$

$$C = 90° - B$$

$$c: \begin{cases} \cos B = \frac{c}{a} & c = a \cdot \cos B \\ & c = \sqrt{a^2 - b^2} \end{cases}$$



Example: Solve a right triangle knowing a = 415 m and b = 280 m.

sin B = 280/415 = 0.6747; B = arcsin 0.6747 = **42° 25'**

C = 90° - 42° 25' = **47° 35**'

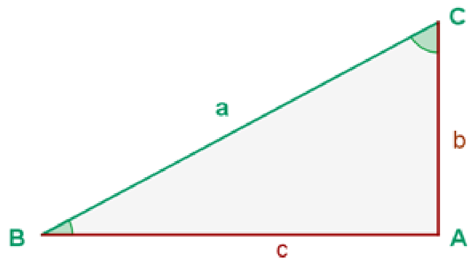
c = a·cos B; c = 415·0.7381 = **306. 31 m**

2nd CASE: two legs are known:

$$B: \ \tan B = \frac{b}{c} \qquad B = \arctan \frac{b}{c}$$

$$C = 90^\circ - B$$

$$a: \begin{cases} \sin B = \dfrac{b}{a} \quad a = \dfrac{b}{\sin B} \\ a = \sqrt{b^2 + c^2} \end{cases}$$



Example: Solve a right triangle knowing b = 33 m and c = 21 m.

tan B = 33/21 = 1.5714; **B = 57° 32**'

C = 90° - 57° 32' = **32° 28'**

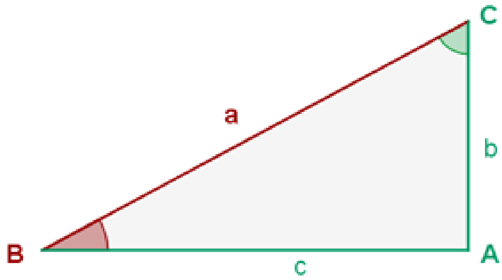
a = b/sin B; a = 33/0.8347 = **39.12 m**

3rd CASE: the hypotenuse and an acute angle are known:

$$C = \ 90^\circ - B$$

$$b: \ \sin B = \frac{b}{a} \qquad b = a \cdot \sin B$$

$$c: \begin{cases} \cos B = \dfrac{c}{a} \quad c = a \cdot \cos B \\ c = \sqrt{a^2 - b^2} \end{cases}$$



Example: Solve a right triangle knowing a = 45 m y B = 22°.

C = 90° – 22° =**68°**

b = a·sin 22°; b = 45·0.3746 =**16.85 m**

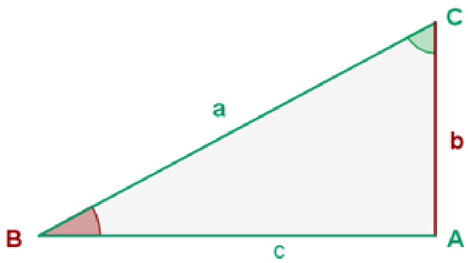
c = a·cos 22°; c = 45·0.9272 =**41.72 m**

4th CASE: a leg and an acute angle are known:

$$C = 90° - B$$

$$a:\ \sin B = \frac{b}{a} \qquad a = \frac{b}{\sin B}$$

$$c:\ \begin{cases} \cot B = \dfrac{c}{b} \quad c = b \cdot \cot B \\ c = \sqrt{a^2 - b^2} \end{cases}$$



Example: Solve a right triangle knowing b = 5.2 m and B = 37°.

C = 90° – 37° = **53°**

a = b/sin B; a = 5.2/0.6018 =**8.64 m**

c = b·cot B; c = 5.2·1.3270 =**6.9 m**

EXERCISE 5:

5.1 The known data for a right triangle ABC is a = 5 m and B = 41.7°. Solve the triangle.

5.2 The known data for a right triangle ABC is b = 3 m and B = 54.6°. Solve the triangle.

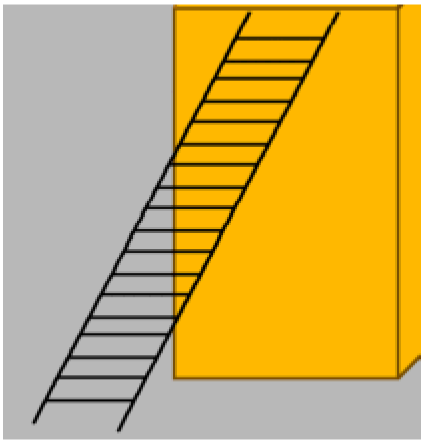
5.3 The known data for a right triangle ABC is a = 6 m and b = 4 m. Solve the triangle.

5.4 The known data for a right triangle ABC is b = 3 m and c = 5 m. Solve the triangle.

5.5 A tree which is 50 m tall casts a shadow which is 60 m long. Find the angle of elevation of the sun at that time.

5.6 An airplane is flying at an altitude of 800 m when it spots a village in the distance with a depression angle of 12°. How far is the village from the point on the ground which the plane is flying over?

5.7 A six-meter-long ladder is leaning against a building. If the ladder makes an angle of 60° with the ground, how far up the wall does the ladder reach? How far from the wall is the base of the ladder?

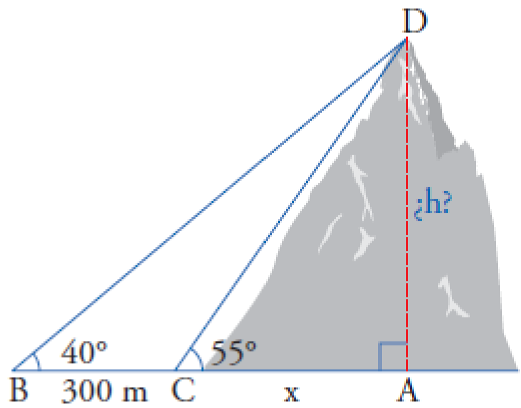


5.8 An isosceles triangle has two sides which are 6 cm long, and the angle between these sides is 42°. Calculate the length of the unequal side and its area.

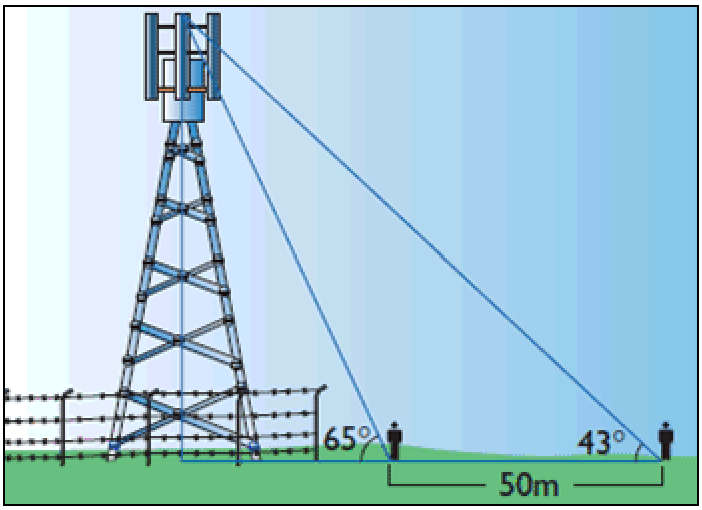
5.9 Two men on opposite sides of a TV tower whose height is 24 m notice the angle of elevation of the top of this tower are 45° and 60° respectively. Find the distance between the two men.

5.10 Find the radius of a circle knowing that a chord whose length is 24.6 m has a corresponding arc of 70°.

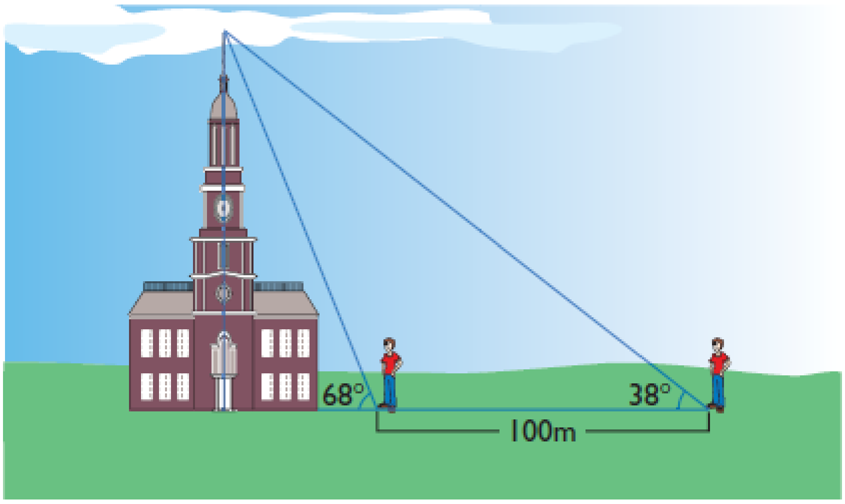
5.11 We measure from a point on the plain the B elevation angle, and we get 40°. Then we walk 300 m towards the mountain, we measure the C elevation angle again, and we get 55°. Calculate the height of the mountain.



5.12 A mobile phone transmitter is on a plain inside an enclosure which we cannot enter. To find out its height, we measure from outside the elevation angle and we get a result of 65°. We walk away 50 m and the new elevation angle is 43°. Calculate the height of the transmitter.



5.13 To measure the height of a building, we measure the elevation angle of the highest point from a certain place, and we get 68°; then we walk away 100 m in the same direction, and the new elevation angle is 38°. Find out the height of the building.

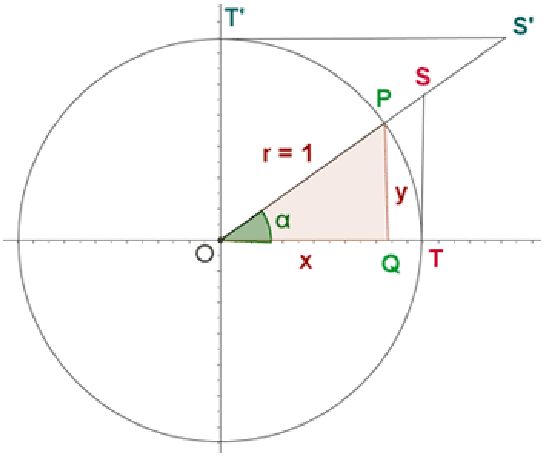


7 UNIT CIRCLE

The **unit circle** has its **centre at (0, 0)** and its **radius is one**.

In the **unit circle**, the coordinate axes delimit four quadrants that are numbered in an anti-clockwise direction.

By drawing a triangle on the **unit circle**, we can draw straight lines called **line values**, whose **length** represents the value of a **trigonometric function**.



QOP and T'OS' are similar triangles.

$$\sin \alpha = \frac{PQ}{OP} = \frac{PQ}{r} = PQ \qquad \csc \alpha = \frac{OP}{PQ} = \frac{OS'}{OT'} = \frac{OS'}{r} = OS'$$

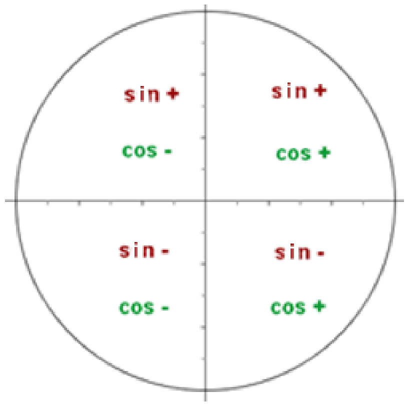
$$\cos \alpha = \frac{OQ}{OP} = OQ \qquad \sec \alpha = \frac{OP}{OQ} = \frac{OS}{OT} = \frac{OS}{r} = OS$$

$$\tan \alpha = \frac{PQ}{OQ} = \frac{ST}{OT} = \frac{ST}{r} = ST \qquad \cot \alpha = \frac{OQ}{PQ} = \frac{S'T'}{OT'} = \frac{S'T'}{r} = S'T'$$

The **sine** is the **ordinate**, that is to say, the **y-coordinate** or the **vertical** value in a pair of coordinates.

The **cosine** is the **abscissa**, that is to say, the **x-coordinate** or the **horizontal** value in a pair of coordinates.

From this data, the **signs of the trigonometric** ratios in each quadrant are as follows:



α :	0°	90°	180°	270°
sin	0	1	0	−1
cos	1	0	−1	0
tg	0	$\to\infty$	0	$\to -\infty$

Examples:

A) Knowing that sin α = 3/5, and 90° < α <180°, calculate the remaining **trigonometric ratios** of angle α.

$$\sin\alpha = \frac{3}{5} \qquad \csc\alpha = \frac{5}{3}$$

$$\cos\alpha = -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\frac{4}{5} \qquad \sec\alpha = -\frac{5}{4}$$

$$\tan\alpha = -\frac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4} \qquad \cot\alpha = -\frac{4}{3}$$

B) Knowing that tan α = 2, and 180° < α <270°, calculate the remaining trigonometric ratios of angle α.

$$\sec\alpha = -\sqrt{1+4} = -\sqrt{5} \qquad \cos\alpha = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\sin\alpha = 2\cdot\left(-\frac{\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5} \qquad \csc\alpha = -\frac{\sqrt{5}}{2}$$

$$\tan\alpha = 2 \qquad \cot\alpha = \frac{1}{2}$$

EXERCISE 6:

6.1 Suppose that cos α = -3/5 and α is in quadrant II. Find the other trigonometric ratios for α .

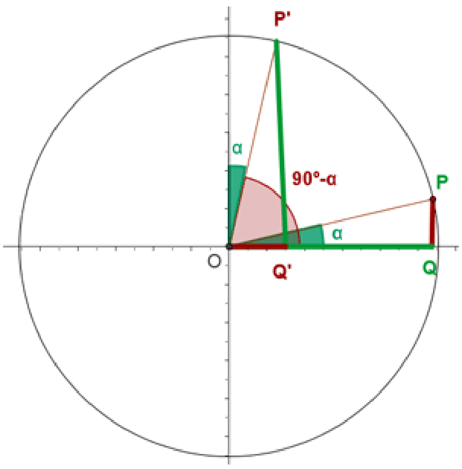
6.2 Suppose that sin $\alpha = \frac{\sqrt{2}}{5}$ and α is in quadrant I. Find the other trigonometric ratios for α.

6.3 Knowing that cos $\alpha = \frac{1}{4}$, and that $270° < \alpha < 360°$, calculate the remaining trigonometric ratios of angle α.

8 TRIGONOMETRIC IDENTITIES

8.1 COMPLEMENTARY ANGLES

Two angles are **complementary** if their sum is 90° or $\pi/2$ radians.



$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

Example:

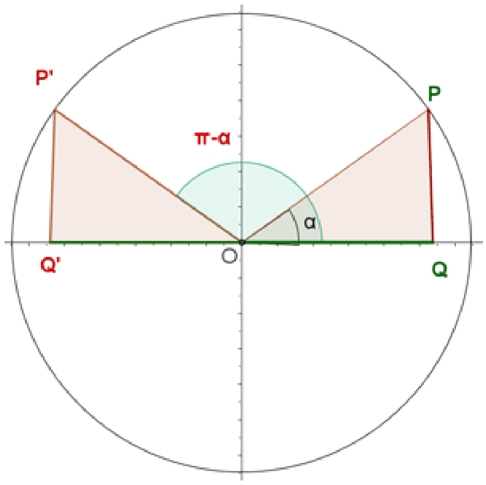
$$sen\ 60° = sen\ (90° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \cos\ (90° - 30°) = sen\ 30° = \frac{1}{2}$$

$$tg\ 60° = tg\ (90° - 30°) = cotg\ 30° = \sqrt{3}$$

8.2 SUPPLEMENTARY ANGLES

Two angles are **supplementary** if their sum is 180° or π radians. So, if two supplementary angles are added, a straight angle is obtained.



$$\sin(\pi-\alpha)=\sin\alpha$$
$$\cos(\pi-\alpha)=-\cos\alpha$$
$$\tan(\pi-\alpha)=-\tan\alpha$$

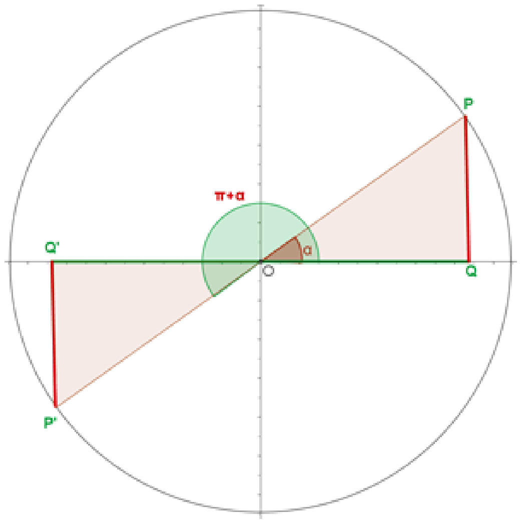
Example:

$$\sin 150^\circ=\sin(180^\circ-30^\circ)=\sin 30^\circ=\frac{1}{2}$$

$$\cos 150^\circ=\cos(180^\circ-30^\circ)=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$$

$$\tan 150^\circ=tg(180^\circ-30^\circ)=-\tan 30^\circ=-\frac{\sqrt{3}}{3}$$

8.3 ANGLES THAT DIFFER BY 180° OR π RADIANS



$$\sin(\pi+\alpha)=-\sin\alpha$$
$$\cos(\pi+\alpha)=-\cos\alpha$$
$$\tan(\pi+\alpha)=\tan\alpha$$

Example:

$$\sin 210^\circ=sen(180^\circ+30^\circ)=-\sin 30^\circ=-\frac{1}{2}$$

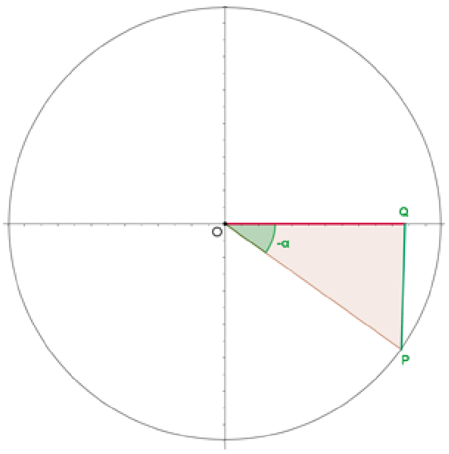
$$\cos 210^\circ=\cos(180^\circ+30^\circ)=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$$

$$\tan 210^\circ=\tan(180^\circ+30^\circ)=\tan 30^\circ=\frac{\sqrt{3}}{3}$$

8.4 EVEN-ODD IDENTITIES

An angle is negative if it moves in the same direction as a clock.

$-\alpha = 360° - \alpha$



$$\text{sen } (-\alpha) = -\text{sen } \alpha$$

$$\cos (-\alpha) = \cos \alpha$$

$$\text{tg } (-\alpha) = -\text{tg } \alpha$$

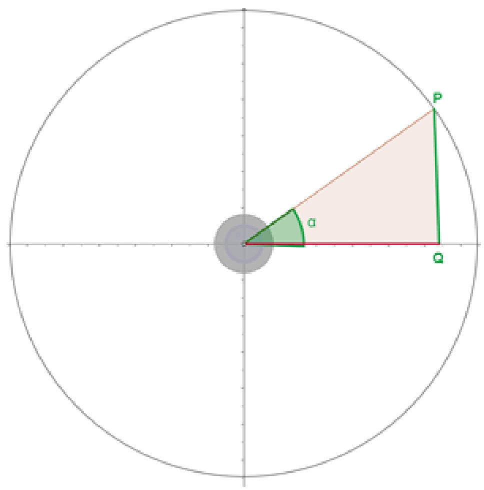
Example:

$$\text{sen } (-30°) = -\text{sen } 30° = -\frac{1}{2}$$

$$\cos (-30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{tg } (-30°) = -\text{tg } 30° = -\frac{\sqrt{3}}{3}$$

8.5 ANGLES GREATER THAN 360°



$$\sin (\alpha + 2\pi k) = \sin \alpha$$

$$\cos (\alpha + 2\pi k) = \cos \alpha$$

$$\tan (\alpha + 2\pi k) = \tan \alpha$$

Example:

$$\begin{array}{r|l} 750° & 360° \\ \hline 30° & 2 \end{array}$$

$$\sin 750° = \sin (360° \cdot 2 + 30°) = \sin 30° = \frac{1}{2}$$

$$\cos 750° = \cos (360° \cdot 2 + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 750° = \tan (360° \cdot 2 + 30°) = \tan 30° = \frac{\sqrt{3}}{3}$$

EXERCISE 7:

7.1 Calculate the trigonometric ratios of the following angles:

- **a)** 225°
- **b)** 330°
- **c)** 2655°
- **d)** –840°
- **e)** –150°

7.2 Suppose that sin 20° = 0.3420 and cos 20° = 0.9397. Calculate cos 70°, sin 70°, tg 20° and tg 70°.

7.3 Find the exact value of sin 315°, cos 315° and tg 315°.

7.4 Find the exact value of sin 210°, cos 210° and tg 210°.

7.5 Find the exact value of sin 135°, cos 135° and tg 135°.

7.6 Suppose that sin 15° = 0.2588. Calculate without using the calculator these trigonometric ratios:

a) sin 165° b) sin 195°

c) sin 345° d) cos 105°

e) cos 255° f) cos 285°