UNIT 1. NUMBERS

1. REAL NUMBERS.

This classification of numbers represents the most accepted elementary classification, and it is useful in computing science.

Class	Symbol	Description
Natural Number	$\mathbb{N}$	Natural numbers are defined as non-negative counting numbers: $\mathbb{N} = \{ 0, 1, 2, 3, 4, ... \}$. Some exclude 0 (zero) from the set: $\mathbb{N}^* = \mathbb{N} \setminus \{0\} = \{ 1, 2, 3, 4, ... \}$.
Integer	$\mathbb{Z}$	Integers extend $\mathbb{N}$ by including the negative of counting numbers: $\mathbb{Z} = \{ ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ... \}$. The symbol $\mathbb{Z}$ stands for Zahlen, the German word for "numbers".
Rational Number	$\mathbb{Q}$	A rational number is the ratio or quotient of an integer and another non-zero integer: $\mathbb{Q} = \{n/m \mid n, m \in \mathbb{Z}, m \neq 0 \}$. E.g.: -100, -20¼, -1.5, 0, 1, 1.5, 1½ 2¾, 1.75, etc
Irrational Number		Irrational numbers are numbers which cannot be represented as fractions. E.g.: $\sqrt{2}$, $\sqrt{3}$, π, e.
Real Number	$\mathbb{R}$	Real numbers are all numbers on a number line. The set of $\mathbb{R}$ is the union of all rational numbers and all irrational numbers.
Imaginary Number		An imaginary number is a number whose square is a negative real number, and it is denoted by the symbol i, so that $i^2 = -1$. E.g.: -5i, 3i, 7.5i, etc. In some technical applications, j is used as the symbol for imaginary number instead of i.
Complex Number	$\mathbb{C}$	A complex number consists of two parts, a real number and an imaginary number, and it is also expressed in the form a + bi (i is notation for the imaginary part of the number). E.g.: 7 + 2i

❖ Rational Numbers

A **Rational** Number can be written as a **Ratio** of two integers (i.e. a simple fraction).

Example: **1.5** is rational, because it can be written as the ratio **3/2**

Example: **7** is rational, because it can be written as the ratio **7/1**

Example **0.317** is rational, because it can be written as the ratio **317/1000**

❖ Irrational Numbers

Some numbers **cannot** be written as a ratio of two integers... so they are called **Irrational Numbers**.

?	*They are called **irrational** because they cannot be written as a **ratio** (or fraction), not because they are crazy!*

Example: **π** **(Pi)** (pronounced /paɪ/) is an irrational number.

	**π = 3.14159265358979323846264338327950 (and more...)** You **cannot** write down a simple fraction that equals Pi.

The popular approximation of $^{22}/_{7}$ = 3.1428571428571... is close but **not accurate**.

Another clue is that the decimal goes on forever without repeating.

Rational vs Irrational

So you can tell the difference between Rational and Irrational by trying to write the number as a simple fraction.

Example: 9.5 can be written as a simple fraction like this:

9.5 = $^{19}/_{2}$

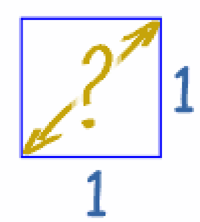
So it is a **rational number** (and so is **not irrational**)

Here are some more examples:

Number	As a Fraction	Rational or Irrational?
5	5/1	Rational
1.75	7/4	Rational
.001	1/1000	Rational
√2 (square root of 2)	?	**Irrational !**

Square Root of 2

Let's look at the square root of 2 more closely.

	If you draw a square (of size "1"), what is the distance across the diagonal?

The answer is the **square root of 2**, which is **1.4142135623730950... (etc)**

But it is not a number like 3, or five-thirds, or anything like that; in fact, you **cannot** write the square root of 2 using a ratio of two numbers... and so we know it is **an irrational number.**

If you would like to know **why**, visit this webpage:

http://www.mathsisfun.com/numbers/irrational-finding.html

Famous Irrational Numbers

<table>
<tr><td></td><td>Pi /paɪ/ is a famous irrational number. Pi has been calculated to over one million decimal places and still there is no repeating pattern. The first few digits look like this:
3.1415926535897932384626433832795 (and more ...)</td></tr>
<tr><td></td><td>The number e (Euler's Number) is another famous irrational number. e has also been calculated to lots of decimal places without any pattern showing. The first few digits look like this:
2.7182818284590452353602874713527 (and more ...)</td></tr>
<tr><td></td><td>The Golden Ratio is an irrational number. The first few digits look like this:
1.61803398874989484820... (and more ...)

Interesenting videos:
http://www.youtube.com/watch?v=085KSyQVb-U
http://www.youtube.com/watch?v=PNQk_GJuZQo
http://www.youtube.com/watch?v=2zWivbG0RIo</td></tr>
<tr><td></td><td>Many square roots, cube roots, etc are also irrational numbers. Examples:<table><tr><td>√3</td><td>1.7320508075688772935274463415059 (etc)</td></tr><tr><td>√99</td><td>9.9498743710661995473447982100121 (etc)</td></tr></table></td></tr>
<tr><td colspan="2">But √4 = 2 (rational), and √9 = 3 (rational) ...
... so not all roots are irrational.</td></tr>
</table>

History of Irrational Numbers

Apparently Hippasus (one of Pythagoras' students) discovered irrational numbers when trying to represent the square root of 2 as a fraction (using geometry, it is thought). Instead he proved that one cannot write the square root of 2 as a fraction and so it was irrational.

However, Pythagoras could not accept the existence of irrational numbers, because he believed that all numbers had perfect values. But he could not disprove Hippasus' "irrational numbers", and so Hippasus was literally thrown overboard[1] and drowned in the sea!

[1] overboard = por la borda

Video about the history of mathematics:
http://www.youtube.com/watch?v=cy-8lPVKLIo

Video about the classification of real numbers:
http://www.youtube.com/watch?v=7I2C9BYb7pk

<u>QUESTION 1.</u> Which one of the following is not an irrational number? $\sqrt{2}$, $\sqrt{4}$, $\sqrt{3}$, $\sqrt{5}$.

<u>QUESTION 2.</u> Draw a classification of numbers using the following diagrams:

Diagram 1:

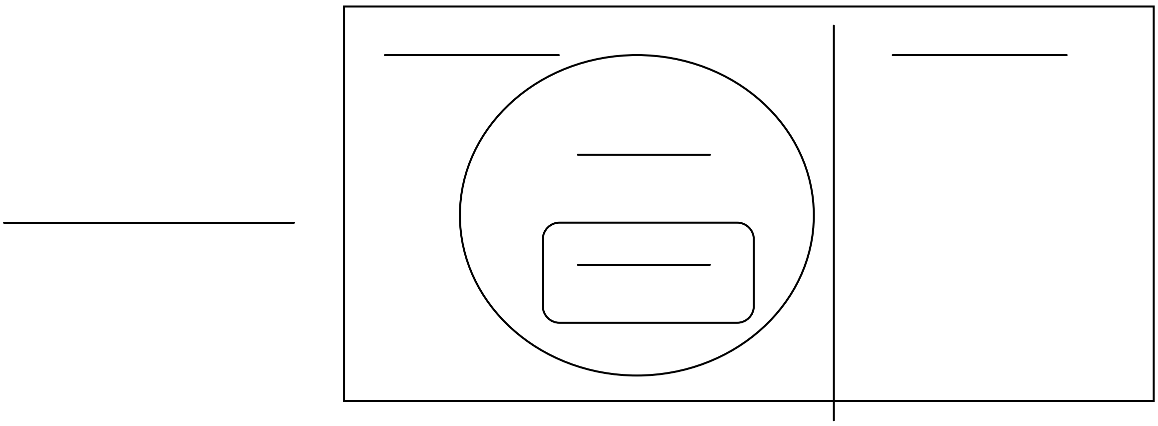
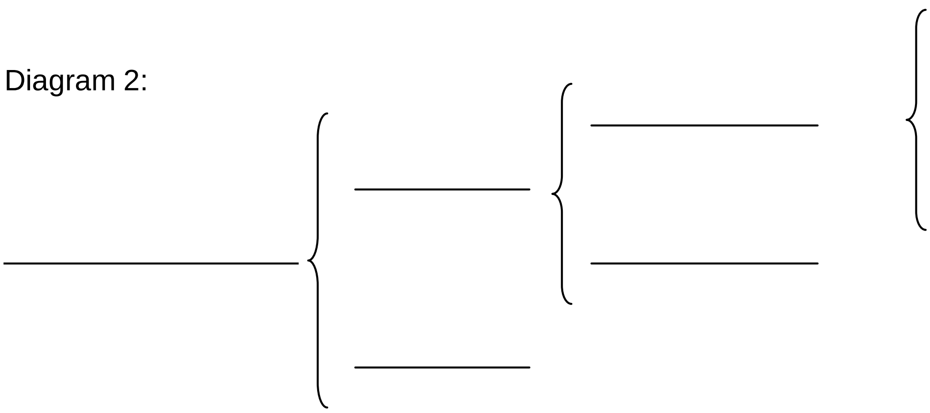


Diagram 2:



Representation of real numbers:

Representation of rational numbers:
http://www.youtube.com/watch?v=axtnjl_tqlg

Representation of numbers in the number line:
http://www.youtube.com/watch?v=TGUQtv4O2j4

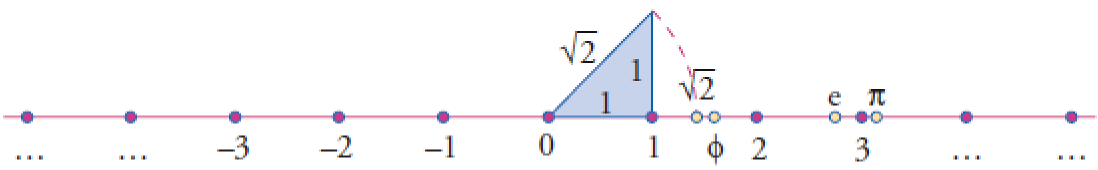
Example 1: Represent the number 3/5

Example 2: Represent the number 7/ 5

Example 3: Represent the number 13/4

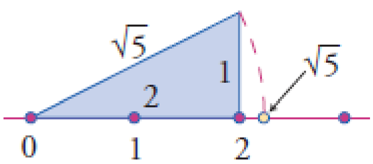
Example 4: Represent the number - 8/3

Solved example 5: Represent the number $\sqrt{2}$.



$$\sqrt{5} = \sqrt{2^2 + 1^2}$$

Solved example 6: Represent the number $\sqrt{5}$:



Example 7: Represent the number $\sqrt{8}$

Example 8: Represent the number $\sqrt{10}$

2. ROUNDING NUMBERS.

What is "Rounding"? *Rounding means reducing the digits in a number while trying to keep its value similar.* The result is less accurate, but easier to use.

Example: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

Common Method

There are several different methods for rounding, but here we will only look at the **common method**, the one used by most people.

How to Round Numbers

- Decide which is the last digit to **keep.**
- Leave it the same if the **next digit** is less than 5 (this is called *rounding down*).
- But increase it by 1 if the next digit is 5 or more (this is called *rounding up*).

Example: Round 74 to the nearest 10

- We want to keep the "7" as it is in the 10s position.
- The next digit is "4" which is less than 5, so we do not need to change "7".

So, the answer is 70 .

(74 gets "rounded down")

Example: Round 86 to the nearest 10

- We want to keep the "8".
- The next digit is "6" which is 5 or more, so we increase the "8" by 1 to "9".

So the answer is 90.

(86 gets "rounded up")

So: when the first digit **removed** is 5 or more, we increase the last digit **remaining** by 1.

More examples: Rounding to the hundredths (two decimal numbers)

a) $4{,}52385 = 4{,}52$ b) $3{,}34813 = 3{,}35$

c) $0{,}62543 = 0{,}63$ d) $18{,}59732 = 18{,}60$

WARNING: In this case we can say that we are rounding to three significant digits.

2.1. ABSOLUTE ERROR.

Every time we round a number, we are making a mistake – an error.
The **absolute error** is the absolute value of the difference between the exact value and the rounded value.

EXAMPLE: If we round $\sqrt{5} = 2{,}236067\ldots$ to two decimal numbers (to the hundredths) we get 2.24, so the absolute error is:

$$|\sqrt{5} - 2{,}24| = |2{,}236067\ldots - 2{,}24| = |-0{,}003932\ldots| = 0{,}003932\ldots$$

2.2. RELATIVE ERROR.

The relative error is the result of this division: $\dfrac{absolute\ \ error}{exact\ \ value}$.

Solved example: Calculate the absolute error and relative error if we round $\sqrt{3}$ to the hundredths (to two decimal numbers)

$$\sqrt{3} = 1{,}73205\ldots \approx 1{,}73$$

Absolute error = $|\sqrt{3} - 1{,}73| = 0{,}00205\ldots$

Relative error = $\dfrac{0{,}00205\ldots}{1{,}73205\ldots} = 0{,}00118$

Exercise: Calculate absolute error and relative error if we round $\sqrt{7}$ to the hundredths (to two decimal numbers).

3. INTERVALS.

3.1. Definition: An interval is all the numbers between two given numbers.

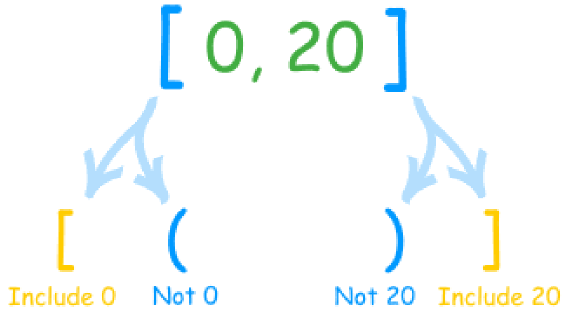
Example: (2,4) → the interval 2 to 4 includes all numbers bigger than 2 and smaller than 4.

3.2. Interval Notation

In "Interval Notation" you just write the beginning and ending numbers of the interval, and use:

- [ ] a square bracket if you want to **include** the end values, or
- ( ) a round bracket if you **don't.**

Like this:



Example: (5, 12]

Means from 5 to 12, do **not** include 5, but **do** include 12.

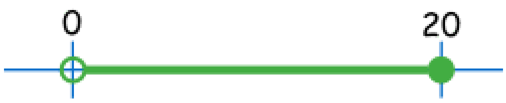
3.3. Number Line

With the Number Line you draw a thick line to show the values you are including, and:

- a filled-in circle if you want to include the end value, or
- an open circle if you don't

Like this:

Example:

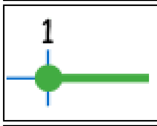
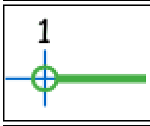
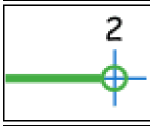
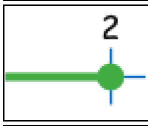


means all the numbers between 0 and 20, do **not** include 0, but **do** include 20…

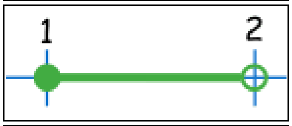
…or in interval notation: (0, 20].

3.4. All Three Methods Together

Here is a handy table showing you all 3 methods (the interval is 1 to 2):

	From 1			**To 2**	
	Including 1	**Not Including 1**		**Not Including 2**	**Including 2**
Inequality:	x ≥ 1 "greater than or equal to"	x > 1 "greater than"		x < 2 "less than"	x ≤ 2 "less than or equal to"
Number line:					
Interval notation:	[1	(1		2)	2]

Example: to **include 1**, and **not include 2** we would have:

Inequality:	x ≥ 1 and x < 2 or together: 1 ≤ x < 2
Number line:	
Interval notation:	[1, 2)

3.4. Open or Closed

The terms "Open" and "Closed" are sometimes used when the end value is included or not:

(a, b)	**a < x < b**	**an open interval**
[a, b)	**a ≤ x < b**	**closed on the left, open on the right**
(a, b]	**a < x ≤ b**	**open on the left, closed on the right**
[a, b]	**a ≤ x ≤ b**	**a closed interval**

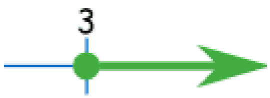
These are intervals of finite length. There are also have intervals of infinite length.

3.5. To Infinity (but not beyond!)

We often use Infinity in interval notation.

Infinity is **not a real number**, in this case it just means "continuing on and on..."

Example: x greater than, or equal to, 3: [3, +∞)



Note that we use the round bracket with infinity, because we never reach it!

There are 4 possible "infinite end" intervals:

Interval	Inequality	
(a, +∞)	x > a	"greater than a"
[a, +∞)	x ≥ a	"greater than or equal to a"
(-∞, a)	x < a	"less than a"
(-∞, a]	x ≤ a	"less than or equal to a"

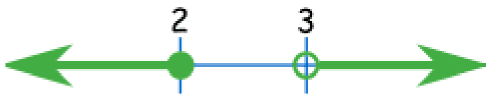
We could even show **no limits** by using this notation: (-∞, +∞)

3.6. Two Intervals

We can have two (or more) intervals.

Example: x ≤ 2 or x >3

On the number line it would look like this:



Interval notation would look like this: **(-∞, 2] U (3, +∞)**

We used a "U" to mean Union (the joining together of two sets).

3.7. Union and Intersection

We have just seen how to join two sets using "Union" (and the symbol ∪).

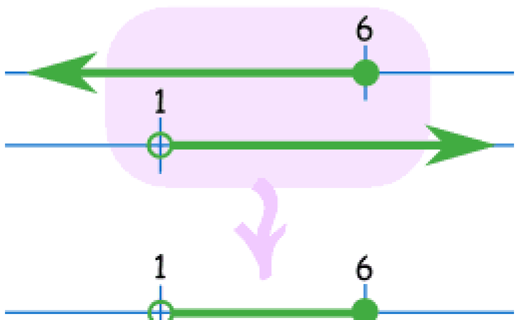
There is also "Intersection" which means "has to be in both". *Think: "where do they overlap?"*

The Intersection symbol is an upside down "U" like this: ∩

Example: (-∞, 6] ∩ (1, ∞)

The first interval goes up to (and including) 6

The second interval goes from (but not including) 1 onwards.



The **Intersection** (or overlap) of those two sets goes from 1 to 6 (not including 1, including 6): (1, 6]

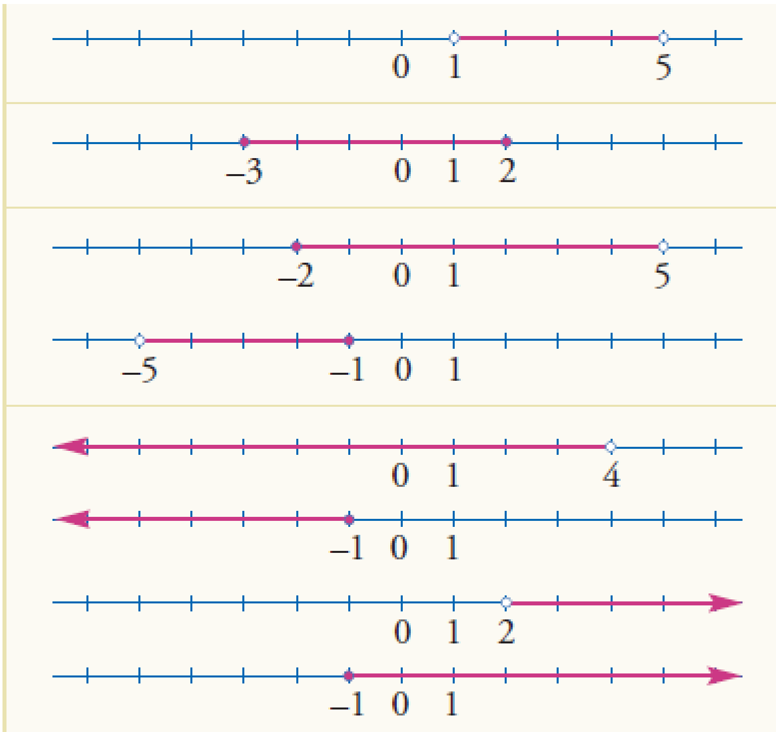
Conclusion

- **An Interval is all the numbers between two given numbers.**
- **It is important to show whether or not the beginning and end number are included.**
- **There are three main ways to show intervals: Inequalities, The Number Line and Interval Notation.**

EXERCISES. To check if you've understood what intervals mean, try to answer the following questions:

http://www.mathopolis.com/questions/q.php?id=451&site=1&ref=/sets/intervals.html&qs=451_454_1073_1074_1075_1076_2316_2317_2318_2319

Exercise 2:

Interval	Representation
	

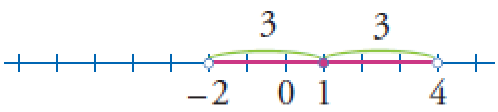
3.8. **Ranges** (in Spanish: *entornos*)

A range with centre ***a*** and radius ***r*** is the set of numbers whose distance to ***a*** is smaller than the radius ***r*** (so, it is an open interval):

$$E(a, r) = \{x \in \mathbb{R};\ d(x, a) < r\} = \{x \in \mathbb{R};\ |x - a| < r\} = (a - r, a + r)$$

Example:

$$E(1, 3) = \{x \in \mathbb{R};\ d(x, 1) < 3\} = \{x \in \mathbb{R};\ |x - 1| < 3\} = (-2, 4)$$



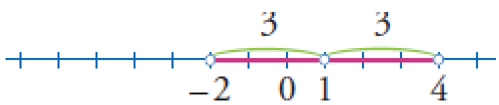
Example: Represent E (2, 4).

A **range is reduced** when a centre ***a*** radius ***r*** is a range but the centre is not included (in other words, a range is reduced when its centre is not included).

Example:

$$E^*(a, r) = E(a, r) - \{a\}$$

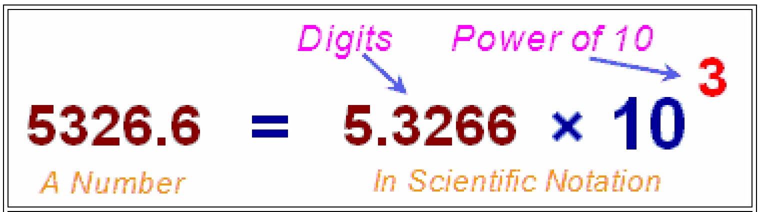
$$E^*(1, 3) = (-2, 4) - \{1\}$$



4. SCIENTIFIC NOTATION / STANDARD FORM

Scientific Notation (also called Standard Form) consists in writing a number in two parts, like this:

- Just the **digits** (with the decimal point placed after the first digit), followed by
- **× 10 to a power** that would put the decimal point back where it should be (i.e. it shows how many places to move the decimal point).



In this example, 5,326.6 is written as **5.3266 × 10^3**, because 5,326.6 = 5.3266 × 1,000 = 5,326.6 × 10^3

Try it yourself:

http://www.mathsisfun.com/numbers/scientific-notation.html

More examples of scientific notation:

http://www.mathsisfun.com/numbers/scientific-notation.html

Why Use Scientific Notation?

Because it makes it easier when you are dealing with very big or very small numbers, which are common in scientific and engineering work.

For example, it is easier to write (and read) 1.3×10^{-9} than 0.0000000013.

And it can make calculations easier, as in this example:

Example: a tiny space inside a computer chip has been measured to be 0.00000256 m wide, 0.00000014 m long and 0.000275 m high.

What is its volume?

Let's first convert the three lengths into scientific notation:

- width: 0.000 002 56 m = 2.56×10^{-6}
- length: 0.000 000 14 m = 1.4×10^{-7}
- height: 0.000 275 m = 2.75×10^{-4}

Then multiply the digits together (ignoring the ×10s):

$$2.56 \times 1.4 \times 2.75 = 9.856$$

Last, multiply the ×10s:

$10^{-6} \times 10^{-7} \times 10^{-4} = 10^{-17}$ (this was easy: I just added -6, -4 and -7 together)

The result is $\mathbf{9.856\times10^{-17}\ m^3}$

Engineering Notation

Engineering Notation is like Scientific Notation, except that you only use powers of ten that are multiples of 3 (such as 10^3, 10^{-3}, 10^{12} etc).

Example: 19,300 would be written as $\mathbf{19.3 \times 10^3}$

Example: 0.00012 would be written as $\mathbf{120 \times 10^{-6}}$

Notice that the "digits" part can now be between 1 and 999 (it can be 1, but never 1,000).

The advantage is that you can replace the **×10**s with Metric Numbers, so you can use standard words (such as thousand or million) prefixes (such as kilo, mega) or a symbol (k, M, etc)

Example: 19,300 meters would be written as $\mathbf{19.3 \times 10^3}$ **m, or 19.3 km**

Example: 0.00012 seconds would be written as $\mathbf{120 \times 10^{-6}}$ **s, or 120 microseconds**

Another example: $3{,}5 \cdot 10^{8} : (2{,}5 \cdot 10^{-5}) = 1{,}4 \cdot 10^{13}$

Now, using the calculator: $\boxed{3.5}\ \boxed{\text{EXP}}\ \boxed{8}\ \boxed{\div}\ \boxed{2.5}\ \boxed{\text{EXP}}\ \boxed{-5}\ \boxed{=}\ \boxed{1{,}4\times10^{13}}$

5. FACTORIAL FUNCTION.

The **factorial function** (symbol: **!**) just means we multiply a series of descending natural numbers. Examples:

- **4!** = 4 × 3 × 2 × 1 = 24
- **7!** = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040
- **1!** = 1

The factorial function is useful, for example, to calculate the number of combinations of a given number of elements. For example, with six letters you can generate 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720 different passwords without repeating any letters (because the position of the elements is important, each password is not really a different combination, but a different permutation).

"4!" is usually pronounced "4 factorial" (some people even say "4 shriek" or "4 bang", because these words sound like exclamations).

Question: What is 10! if you know that 9! = 362,880 ?

What About "0!"

Zero Factorial is interesting... it is generally agreed that **0! = 1**. (if you don't use any letters to write a password, you will only have one possibility: write nothing, and press ENTER!)

Where is Factorial Used?

Factorials are used in many areas of mathematics, but particularly in Combinations and Permutations, especially when working with statistical and probability questions. (a combination is just a permutation in which the order of the elements is not important).

Example: What is 7! / 4!

Let's write them out in full:

$$\frac{\mathbf{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\mathbf{4 \times 3 \times 2 \times 1}} \mathbf{= 7 \times 6 \times 5 = 210}$$

As you can see, the **4 × 3 × 2 × 1** "cancelled out", leaving only **7 × 6 × 5**

5.1. COMBINATORIAL NUMBERS.

Combinatorial number ***m*** over ***p*** is defined as the following formula:

$$\binom{m}{p} = \frac{m!}{p!(m-p)!}$$

Example:

$$\binom{8}{3} = \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2 \cdot 1}} = 56$$

and using the calculator $\boxed{8}\ \boxed{\text{nCr}}\ \boxed{3}\ \boxed{=}\ \boxed{56}$

Example: Calculate the following combinatorial numbers:

$\binom{5}{0} =$ $\binom{6}{6} =$ $\binom{8}{1} =$

So, what could we deduce about these combinatorial numbers in general?

$\binom{m}{0} =$ $\binom{m}{m} =$ $\binom{m}{1} =$

Some properties of combinatorial numbers:

a) $\binom{m}{p} = \binom{m}{m-p}$

Example: $\binom{7}{5} = \binom{7}{2}$ Come on, check it!

b) $\binom{m}{p} = \binom{m-1}{p} + \binom{m-1}{p-1}$

Example: $\binom{8}{3} = \binom{7}{3} + \binom{7}{2}$ Come on, check it!

EXERCISES

UNIT 1. NUMBERS.

RATIONAL AND IRRATIONAL NUMBERS

Exercise 1: Convert the following decimal fractions into decimal numbers, and classify these numbers:

a) $\frac{51}{16}$ b) $\frac{36}{11}$ c) $\frac{13}{6}$

Exercise 2: Round the two decimal numbers and calculate:

a) $23{,}567 + 0{,}413 - 12{,}085$

b) $0{,}624 \cdot 1{,}368$

c) $5{,}575 : 8{,}361$

d) $28{,}508 + 12{,}534 : 4{,}197$

Exercise 3: Calculate the absolute error and relative error if you round these numbers to two decimal numbers:

a) 6,4135 b) 0,0785

c) 4,9084 d) 7,0985

Exercise 4: Using scientific notation, calculate:

$a)\ 3\cdot 10^{3}+2\cdot 10^{2}=$

$b)\ 13\cdot 10^{4}+2\cdot 10^{5}=$

$c)\ 5\cdot 10^{5}-25\cdot 10^{4}=$

$d)\ 3.2\cdot 10^{6}\cdot 2\cdot 10^{2}=$

$e)\ 1.2\cdot 10^{-6}\cdot 3\cdot 10^{2}=$

$d)\ 5.2\cdot 10^{7}:2\cdot 10^{-2}=$

$e)\ 2.62\cdot 10^{-24}-7.53\cdot 10^{-23}=$

$f)\ 5.74\cdot 10^{11}+6.5\cdot 10^{12}=$

g) Using the calculator: $3.85\cdot 10^{-15}:(3.5\cdot 10^{-29})=$

Exercise 5: Classify the following numbers:

a) $\sqrt{10}$ b) 2/5

c) $\sqrt{64}$ d) $-\sqrt{50}$

Exercise 6: Represent in the numerical line the following numbers:

a) $\sqrt{13}$

b) $-\sqrt{20}$

Exercise 7: Represent approximately in a numerical line the following numbers:

a) $\sqrt{15}$ b) $-\pi$ c) $\sqrt{23}$ d) $-\sqrt{14}$

Exercise 8: Calculate:

a) $\frac{4}{5} + 3 - \frac{7}{15}$

b) $\frac{1}{6} - \frac{5}{9} \cdot \frac{3}{2}$

c) $\frac{1}{2} : \left(\frac{3}{4} - 1 + \frac{5}{8}\right)$

d) $\frac{3}{5}\left(\frac{1}{3} - 2 + \frac{2}{5}\right)$

INTERVALS

Exercise 9: Write as the following intervals inequalities, classify and represent them in a numeric line:

a) $(-2, 4]$

Classification: ______________________________

Inequality: _________________________________

Representation ____

b) $[-5, 1]$

Classification: ______________________________

Inequality: _________________________________

Representation ____

c) $[3, +\infty)$

Classification: ______________________________

Inequality: _________________________________

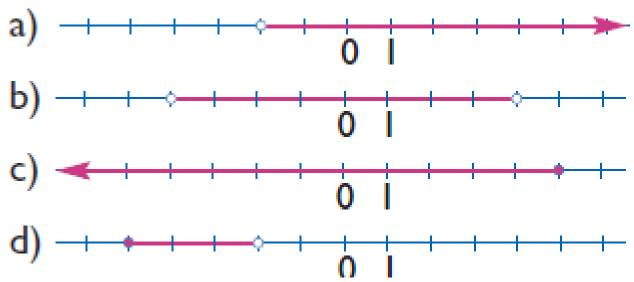
Representation ____

d) $(-\infty, -3)$ Classification: ___________________________

Inequality: ______________________________

Representation ____

Exercise 10: Write the intervals which are represented in the line:



Exercise 11: Represent the following ranges:

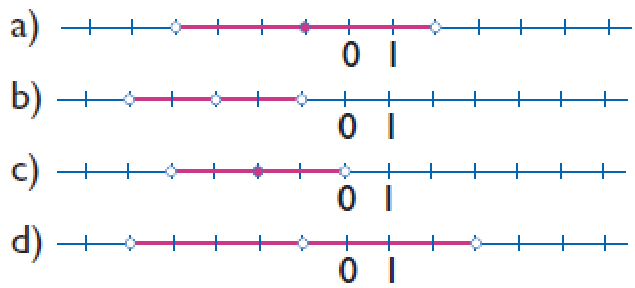
a) $E^*(1, 4)$

b) $E(-1, 2)$

c) $E(-3, 1)$

d) $E^*(0, 3)$

Exercise 12: Write the ranges which are represented in the following numerical lines:



ROUNDINGS AND ERRORS

Exercise 13: Round to two decimal numbers and decide whether it is an excess approximation or a defect approximation:

a) 35/8 b) 13,4972

c) $\sqrt{37}$ d) 2,6283

Exercise 14: Calculate the absolute error and relative error when you round to two decimal numbers:

a) 25/12 b) $\sqrt{8}$

c) 12,3402 d) $\sqrt{80}$

COMBINATORIAL NUMBERS.

Exercise 15: Calculate:

a) $\binom{6}{4}$ b) $\binom{10}{9}$

c) $\binom{40}{40}$ d) $\binom{30}{1}$

Exercise 16: Check whether this equality is true in each case:

$$\binom{m}{p} = \binom{m-1}{p} + \binom{m-1}{p-1}$$

a) $m = 7, p = 3$

b) $m = 10, p = 2$

Exercise 17: Calculate the value of x using the properties of combinatorial numbers:

$$\binom{9}{x-1} = \binom{9}{x-2}$$

MORE EXERCISES

Exercise 18: Write two irrational numbers between 1.5 and 1.7.

Exercise 19: Write the smallest interval whose ends are integers and which contains the golden number, $\phi = \dfrac{1+\sqrt{5}}{2}$.

Exercise 20: Write as an interval:

a) $1 \le x \le 4$ ____________________ b) $x > 2$ ________________

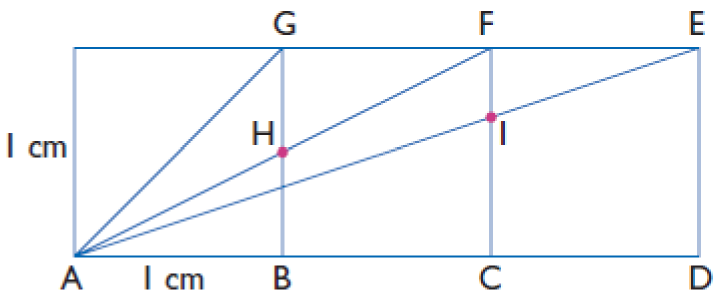
c) $-1 < x \le 5$ ____________________ d) $x < 3$ __________________

Exercise 19: Use the calculator to work out these numbers (three decimal numbers):

a) 2π ________________ b) $\pi + \sqrt{10}$ ________________

c) $\dfrac{1+\sqrt{5}}{2}$ ________________ d) $\sqrt{30} + \sqrt{12}$ ________________

Exercise 20: Look at this drawing, calculate the length of the following segments and classify them as rational or irrational numbers:



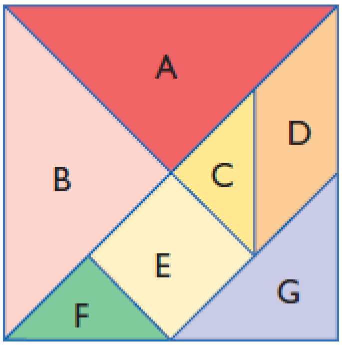
a) BH b) CI

c) AG

d) AF

e) AE

Exercise 21: This figure is called a Chinese Tangram. If the side of the square measures 1 m, calculate the area of every figure which makes up the Tangram.



Exercise 22: Copy the instructions and do the exercises in the ***"Check what you know"*** section on page 25 on your textbook.