Unit 5. POLYNOMIALS.

1. WHAT IS ALGEBRA? WHY STUDY ALGEBRA?

To answer this question, please visit this webpage:

http://math.about.com/od/algebra/a/WhyAlgebra.htm

2. ALGEBRAIC LANGUAGE.

Algebraic language (or algebra language) is a language that uses numbers, letters and brackets, connected with operations. It transmits information. It is used in maths and other sciences, and it replaces natural language.

Example:

NATURAL LANGUAGE	ALGEBRAIC LANGUAGE
In a garden, last year, there were **several** trees, and this year, we have planted **five** more trees. How many trees are there this year? Answer: **5 more than last year**.	x 5 x + 5

Exercise 1:

Haz corresponder cada enunciado con su expresión algebraica:

- La mitad de un número.
- El triple de la mitad de un número.
- La distancia recorrida en x horas por un tren que va a 60 km/h.
- El precio de x kilos de naranjas que están a 1,3 €/kilo.
- La edad de Pedro, sabiendo que su abuelo, que ahora tiene x años, tenía 60 años cuando nació Pedro.
- El área de un triángulo de base 1,3 m y altura x metros.

$1{,}3x$ | $\dfrac{3x}{2}$ | $\dfrac{x}{2}$ | $x - 60$ | $\dfrac{1{,}3x}{2}$ | $60x$

Exercise 2: Complete the table according to the following data:

· Theresa is x years old.

· Her daughter is 25 years younger than her.

· Her mother's age is double her own age.

· Her father is 6 years older than her mother.

· Theresa was 8 years old when her brother Lawrence was born.

	AGE
Theresa	X
Theresa's daughter	
Theresa's mother	
Theresa's father	
Theresa's brother	

Exercise 3:

Lee los enunciados y completa la tabla:

- Eva recibe, de paga semanal, x euros.
- A Leticia le faltan 10 € para recibir el doble que Eva.
- Raquel recibe 50 € más que Leticia.

	PAGA SEMANAL
EVA	x
LETICIA	
RAQUEL	
ENTRE LAS TRES	

Exercise 4. There were f frogs in a pond. 68 of the frogs hopped away. Choose the expression that shows how many frogs are in the pond now.

a) f – 68   b) f + 68   c) f : 68   d) 68 - f   e) 68 + f

3. POLYNOMIALS.

Polynomials are algebraic expressions that include real numbers and variables. Division and square roots cannot be involved in the variables. The variables can only include addition, subtraction and multiplication.

Polynomials contain more than one term. Polynomials are the sums of monomials.

A **monomial** has one term: 5y, or $-8x^2$, or 3.
A **binomial** has two terms: $-3x^2 + 2$, or $-2y^2 + 9y$
A **trinomial** has 3 terms: $-3x^2 + 3x + 2$, or $-2y^2 + 9y + 5$
The **degree** of the term is the exponent of the variable: $3x^2$ has a degree of 2.
When the variable does not have an exponent, we always understand that there's a '1'.

Example: $x^2 - 7x - 6$ (Each part is a term and x^2 is called the leading term.)

Term	Numerical Coefficient
x^2 $-7x$ -6	1 -7 -6

Polynomials are usually written in decreasing order of terms. The largest term or the term with the highest exponent in the polynomial is usually written first. The first term in a polynomial is called the **leading term**. When a term contains an exponent, it tells you the degree of the term.

Here's an example of a three-term polynomial:

$6x^2 - 4xy + 2xy$. → This three-term polynomial has a leading term to the second degree. It is called a second degree polynomial and it is also often called a trinomial.

$9x^5 - 3x^4 - 2x - 2$ →This four-term polynomial has a leading term to the fifth degree and a term to the fourth degree. It is called a fifth degree polynomial.

$3x^3$ → This is a one-term algebraic expression which is called a monomial.

<u>**VERY IMPORTANT**</u>: One thing you will do when solving polynomials is combine like terms (= *términos semejantes*).

Like terms: $6x - 3x = 3x$

Unlike terms: 6xy and 2x or 4.

In this operation the first two terms are like and they can be combined: $5x^2 + 2x^2 = 7x^2$

Exercise 5. Complete this table:

Monomial	$-7x^5$	$4x^3y^2z$	5	$-6x$
Numerical Coefficient				
Degree				

Exercise 6. Complete the following table for this polynomial: $P(x) = 7x^3 - 9x - 2$

Términos	Grado	Coeficientes	Coeficiente principal	Término independiente

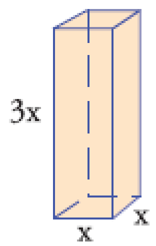
<u>**The numeric value**</u> of an algebraic expression is the value that we get after replacing the variable with a number and doing the operations.

Example: Calculate the numeric value of the algebraic expression **5 x + 3** , when **x = 2:**

$$5 \cdot 2 + 3 = 13$$

The numeric value is **13**.

Exercise 7. Given the figure on the right, calculate its area and its volume as a function of x.



Exercise 8: Order these polynomials according to their degree, from the highest to the lowest. Calculate their leading term and their independent term.

a) $7x^2 - 5x^3 + 4$ b) $-9x^2 - 6x^5 - 7 + 4x^6$

c) $8x^2 - 5x + 4x^5$ d) $-7x^2 - x^8 - 7x + 9 - 4x^6$

Exercise 9: Find the value of **a**, **b** and **c** so that the following polynomials are equal:

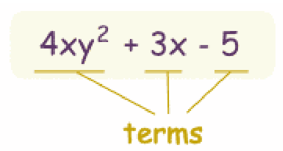
$P(x) = ax^4 - 8x^3 + 4x - b$

$Q(x) = 5x^4 - 8x^3 - cx^2 + 4x + 6$

4. OPERATIONS WITH POLYNOMIALS.

4.1. ADDING AND SUBTRACTING POLYNOMIALS.

A polynomial looks like this:



example of a polynomial;
this one has 3 terms

To **add polynomials** you simply add any like terms together... so, do you remember what like terms are?

Like Terms are terms whose variables (and their exponents such as the ^{2} in x^2) are the same.

In other words, terms that are "like" each other.

Note: the coefficients (the numbers you multiply by, such as "5" in 5x) can be different.

Example: $7x$ and x and $-2x$ are all like terms because the variables are all x.

ADDING POLYNOMIALS:

Do it in two steps:

1st Place like terms together

2nd Add the like terms

Example: Add $2x^2 + 6x + 5$ and $3x^2 - 2x - 1$

Step 1: place like terms together (you can use columns for this):

$$\begin{array}{rrrr} & 2x^2 & +6x & +5 \\ + & 3x^2 & -\ 2x & -1 \\ \hline \end{array}$$

Step 2: add the like terms: $(2+3)x^2 + (6-2)x + (5-1) = 5x^2 + 4x + 4$

Here is an animation to show you:

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html

You can add several polynomials together like that.

Example: Add $2x^2+6x+3xy$, $3x^2-5xy-x$ and $6xy + 5$

SUBTRACTING POLYNOMIALS

To subtract polynomials, first reverse the sign of each term you are subtracting (in other words, turn "+" into "-", and "-" into "+"), then add as usual.

You can also see an animation here:

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html

Copy the example here: ____

Example: Calculate P(x) – Q(x) where

$$P(x) = x^4 - 6x^3 + 7x - 8$$
$$Q(x) = 2x^3 - 3x^2 + 5x - 1$$

:

Exercise 10: Add up the following polynomials:

$$P(x) = 7x^4 - 6x^3 + 5x - 3$$
$$Q(x) = x^4 + 8x^3 - x^2 + 4x + 6$$

Exercise 11: Find the opposite of the following polynomials:

$$P(x) = 5x^5 - 7x^3 + 4x - 1$$
$$Q(x) = -x^4 + 6x^3 - x^2 + 5x + 1$$

Exercise 12: Calculate P(x) – Q(x):

$$P(x) = 5x^4 + x^3 - 2x^2 - 5$$
$$Q(x) = 7x^4 - 5x^2 + 3x + 2$$

Exercise 13: The income and expenses of a company in millions of euros, where t is the number of years the company has been operating, are calculated like this :

$I(t) = t^2 - 3t + 5$

$E(t) = t^2 - 4t + 9$

Find the benefit B(t).

4.2. MULTIPLYING POLYNOMIALS.

To multiply a polynomial:

1° Multiply each term in one polynomial by each term in the other polynomial

2° Add all the results together, and simplify if needed.

Let's look at the simplest cases first:

1 term × 1 term (monomial times monomial)

To multiply one term by another term, first multiply the constants, then multiply each variable together and combine the result:

Example: Multiply $3x^2$ and $2x$:

$3 \cdot 2 \cdot x^2 \cdot x = 6x^3$

Example: Multiply the polynomials P(x) and Q(x), where:

$$P(x) = 2x^3 - 3x^2 + 5$$
$$Q(x) = x^2 - 4x + 6$$

Other examples:

http://www.mathsisfun.com/algebra/polynomials-multiplying.html

Copy some of them here: ____

Exercise 14: Multiply these polynomials:

a) $P(x) = 2x^3 - 3x + 5$

$Q(x) = 3x^2 + x - 4$

b) $P(x) = x^4 - 3x^2 + x - 5 \qquad Q(x) = 2x^3 + x^2 - 4$

c) $P(x) = 3x^5 - x^3 - 5x + 1 \qquad Q(x) = 2x^4 + 4x^2 - 3$

5. REMARKABLE EXPRESSIONS. (= Igualdades notables)

5.1. ADDITION SQUARED. (= Cuadrado de una suma)

First, think about this: Is $(3 + 4)^2$ equal to $3^2 + 4^2$?

So, $\boxed{(a + b)^2 \neq a^2 + b^2}$.

It is very important to learn this law:

The square of an addition is equal to the sum of the square of the first term, plus two times the first term multiplied by the second term, plus the square of the second term (In Spanish: el cuadrado de una suma es igual al cuadrado del primero, más el doble del primero por el segundo, más el cuadrado del segundo):

$$\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$$

It is very easy to prove it. Try it!

Example: $(x + 5)^2 = x^2 + 10x + 25$ Check it on your own!

5.2. SUBTRACTION SQUARED. (= Cuadrado de una diferencia)

First, think about this: Is $(5 - 3)^2$ equal to $5^2 - 3^2$?

So, $\boxed{(a - b)^2 \neq a^2 - b^2}$

Learn this law:

The square of a subtraction is equal to the sum of the square of the first term **minus** two times the first term multiplied by the second term, plus the square of the second term (In Spanish: el cuadrado de una resta es igual al cuadrado del primero, **menos** el doble del primero por el segundo, más el cuadrado del segundo):

$$(\mathbf{a + b})^2 = \mathbf{a}^2 - \mathbf{2ab} + \mathbf{b}^2$$

Example: $(x - 3)^2 = x^2 - 6x + 9$ Check it!

5.3. ADDITION TIMES SUBTRACTION. (= Suma por diferencia)

A sum multiplied by a subtraction is equal to the square of the first term **minus** the square of the second term (In Spanish: una suma por una diferencia es igual al cuadrado del primero **menos** el cuadrado del segundo:

$$(\mathbf{a + b})\,(\mathbf{a - b}) = \mathbf{a}^2 - \mathbf{b}^2$$

Example: $(x + 7)(x - 7) = x^2 - 49$ Check it!

5.4. FACTORIAL DECOMPOSITION OF A POLYNOMIAL.

The factorial decomposition of a polynomial is an expression as a multiplication of prime factors.

(Remember the definition for numbers).

When the decomposition is easy you can do it in your mind. Look at these examples and think about them:

a) $x^2 + 5x = x(x + 5)$

b) $x^2 + 8x + 16 = (x + 4)^2$

c) $x^2 - 9 = (x + 3)(x - 3)$

d) $25x^2 - 10x + 1 = (5x - 1)^2$

e) $x^2 + x + 1/4 = (x + 1/2)^2$

f) $x^2 - 3 = \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

g) $x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)^2$

Exercise 15: Calculate in your mind:

a) $(x + 2)^0$ b) $(x - 3)^1$ c) $(x - 7)^1$ d) $(2x + 6)^0$

Exercise 16: Work out:

a) $(x + 5)^2$ b) $(x + 3)(x - 3)$

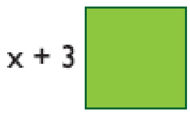
c) $(x - 6)^2$ d) $(x + \sqrt{5})(x - \sqrt{5})$

Exercise 17: Work out and simplify:

a) $(2x + 1/2)^2$ b) $(x/3 + 1)(x/3 - 1)$

c) $(6x - 2/3)^2$ d) $(5x + 3/4)(5x - 3/4)$

Exercise 18: Find a polynomial which expresses the area of the square in this figure:



Exercise 19: Work out the following products:

a) $5x^2(2x^3 - 3x)$ b) $-2x^3(7x^4 - 4x^2)$

c) $-3x(-8x^5 - 5x^2)$ d) $6x^4(-x^5 + 2x)$

Exercise 20: Factorise in your head:

a) $2x^2 + 6x$ b) $x^2 - 6x + 9$

c) $x^2 - 25$ d) $x^2 + 8x + 16$

Exercise 21: Factorise:

a) $12x^4 + 8x^3$ b) $5x^3 + 20x^2 + 20x$

c) $x^2 - 3$ d) $9x^2 - 30x + 25$

PRACTISE MORE EXERCISES

EXERCISE 22. Watch the following video and try to answer the following questions:

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/VRBLC/LO_Template.swf

➔ A) What is algebra?____

____

➔ B) What is a variable?____

____

EXERCISE 23. Watch the following video and then try to answer the following questions:

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/TERLC/LO_Template.swf

You have learnt some new words, let's check them:

- ➔ "Term" is ____________________
- ➔ "Like term" is ____________________
- ➔ "Unlike term" is "____________________
- ➔ "Collect like terms" in Spanish is called " ____________________"

Now, Write an example of like terms:

____

EXERCISE 24. Do the following exercise:

http://webs.ono.com/paco_garces/1ESO/ecuaciones/ejer_ecuac/sumamonomios2.htm

Copy it in your notebook:

EXERCISE 25. Do the following test:
http://www.bbc.co.uk/apps/ifl/schools/ks3bitesize/maths/quizengine?quiz=formulae1&templateStyle=maths

Copy the answer in here. 1. ___ 2. ____ 3.____ 4. ____ 5. ____

EXERCISE 26. Let's revise how to expand brackets.

1. Watch the following video:

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/BRALC/LO_Template.swf

2. Do the following operations with brackets:

a) 5 (3x – 2y +a)=________________________________

b) 3x (2a + 7b – 5c) = ____________________________________

EXERCISE 27. Calculate the following operations with monomials:

a) $7x^5 - 4x^5 + 9x^5$ _____ b) $-5x^2 \cdot x$ _______

c) $(-2x^5)^3$ _______ d) $-6x^3 : (-3x)$ _______

EXERCISE 28. Calculate the following operations with monomials:

a) $(3x^4)^3$ _______ b) $-5x^3 + 2x^3 + 4x^3$ _______

c) $-12x^2 : (-4x)$ _______ d) $-6x^2 \cdot (-9x) \cdot x^3$ _______

EXERCISE 29. Calculate the following operations with monomials:

a) $56x^5 : 8x$ ______ b) $6x^3 \cdot (-9x^2)$ ______

c) $-3x^2 + 15x^2 + 4x^2$ ____ d) $(2x^5)^2$ ______

EXERCISE 30. Calculate:

a) $6x^4 \cdot (-9x^3)$ ________ b) $(-3x^3)^3$ ________

c) $5x - 9x + 7x - x$ ______ d) $6x^5 : 4x$ ________

EXERCISE 31. Calculate the following multiplications with polynomials:

a) $(x^5 - 7x^3 + 6x - 1) \cdot 8x^2$ ______________________________

b) $(2x^4 - 8x^2 + 7x - 9) \cdot 7x^3$ ______________________________

c) $(6x^4 + 5x^3 - 8x + 7) \cdot (-9x)$ ______________________________

d) $(x^4 - 9x^3 + 7x - 6) \cdot (-6x^4)$ ______________________________

EXERCISE 32. Simplify the following algebraic expressions:

a) $8x - 12x^2 + 1 + 7x^2 - 3x - 5$ ______________________________

b) $x^2 - 6x - 5x^2 + 7x^2 - 5x - 9$ ______________________________

c) $-7x - 8 + 9x - 11x^2 + 6 + 8x^2$ ______________________________

d) $7x^2 - 9x + 6 - 7x - 8x^2 + 12$ ______________________________

EXERCISE 33. Expand the brackets and simplify the expressions:

a) $7x - (8x^2 + 9 + 5x^2) - 7x - 2$ ______________________________

b) $2x^2 - 5x - 3(2x^2 + 4x^2 - 5x - 6)$ ______________________________

c) $-(3x - 5 + 9x - 7x^2 + 4) + 10x^2$ ______________________________

d) $7(x^2 - 6x + 9) - 7(3x - 7x^2 + 9)$ ______________________________

EXERCISE 34. Extract common factor:

a) $6x - 8y$ __________ b) $8x^3 - 12x^2$ __________

c) $4x^4 + 10x^3 - 6x^2$ d) $9x^2y + 6xy^2 - 3xy$ __________

EXERCISE 35. Here are two polynomials:

$P(x) = 7x^3 - 4x - 1$ $\qquad$ $Q(x) = -2x^2 + 5x - 3$

a) Add them: ____

b) Subtract them: ____

c) Multiply them: ____

d) What is the degree of the addition P(x) + Q(x) ? ________________________

e) What is the degree of the subtraction P(x) - Q(x)? ________________________

f) What is the degree of the multiplication P(x)* Q(x)? ________________________

EXERCISE 36. Calculate in your mind:

a) $(x + 2)^0$ b) $(x - 2)^0$ c) $(x + 2)^1$ d) $(x - 2)^1$

______ _____ ________ __________

EXERCISE 37. Calculate in your mind using the remarkable expressions:

a) $(x + 2)^2$ b) $(x - 2)^2$ c) $(x + 2)(x - 2)$

________ __________ ____________

EXERCISE 38. Calculate in your mind using the remarkable expressions:

a) $(x + 3)^2$ b) $(x - 3)^2$ c) $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

__________ _________ __________________

EXERCISE 39. Calculate in your mind using the remarkable expressions:

a) $\left(2x + \frac{1}{2}\right)^2$ ____

b) $\left(2x - \frac{1}{2}\right)^2$ ____

c) $\left(2x + \frac{1}{2}\right)\left(2x - \frac{1}{2}\right)$ ______________________________

EXERCISE 40. Replace suspension points with the sign = or $\neq$:

a) $(x-3)^2 \ldots x^2 - 6x + 9$

b) $(x+2)^2 \ldots x^2 + 4$

c) $(x-3)^2 \ldots x^2 - 9$

d) $(x+2)^2 \ldots x^2 + 4x + 4$

EXERCISE 41. Extract the common factor of the following expressions:

a) $8x^2 - 12x$ ____________________

b) $8x^4 + 6x^2$ ____________________

c) $2x^4 + 4x^3 - 6x^2$ ____________________

d) $6x^2y + 4xy^2 - 8xy$

EXERCISE 42. Here are three polynomials. Calculate $P(x) - Q(x) - R(x)$:

$P(x) = 7x^3 - 5x + 1$

$Q(x) = -4x^4 - 9x^2 + 4x - 7$

$R(x) = 5x^4 - 7x^3 + 5x + 6$

6. DIVISION OF POLYNOMIALS.

To explain how to divide polynomials, we will do some examples. The process of dividing polynomials is very similar to the process of dividing numbers.

Solved example 1:

Divide $D(x) = 6x^5 - 30x^3 + 22x^2 + 27x - 11$ into $d(x) = 2x^3 - 4x^2 + 6$

$$
\begin{array}{rrrrrr|l}
\cancel{6x^5} & & -30x^3 & +22x^2 & +27x & -11 & 2x^3 - 4x^2 + 6 \\ \cline{7-7}
-\cancel{6x^5} & +12x^4 & & -18x^2 & & & 3x^2 + 6x - 3 \quad \text{Quotient} \\ \cline{1-4}
 & \cancel{12x^4} & -30x^3 & +4x^2 & +27x & & \\
 & -\cancel{12x^4} & +24x^3 & & -36x & & \\ \cline{2-5}
 & & -\cancel{6x^3} & +4x^2 & -9x & -11 & \\
 & & \cancel{6x^3} & -12x^2 & & +18 & \\ \cline{3-6}
 & & & -8x^2 & -9x & +7 & \text{Remainder}
\end{array}
$$

Just as a matter of interest, English people make divisions writing them this way:

$$x-2\overline{)x^3+x^2-5x-2}$$

When they are dividing $x^3 + x^2 - 5x - 2$ by $x - 2$.

Example 2: Divide $P(x) = 2x^5 - 8x^4 + 12x^2 + 18$ by $Q(x) = x^2 - 3x - 1$.

Example 3: Divide $P(x) = 6x^5 + 2x^4 - 17x^3 + 20x - 25$ into $Q(x) = 2x^3 - 3x + 5$.

Example 4: Divide $P(x) = 2x^3 - 13x + 8$ by $Q(x) = x + 3$.

Do you remember how to check if a division is correct? Do you remember "la prueba de la división"? Write it here:

____

EXERCISE 43: Calculate a polynomial which when divided into $2x^3 - 5x + 1$ has a quotient of $x^2 + 3x - 4$ and a remainder of $-7x^2 + x + 8$.

6.1. RUFFINI'S RULE.

Ruffini's rule has many practical applications; most of them rely on simple division (as demonstrated below) or the common extensions given still further below.

Ruffini´s rule is very useful when in a polynomial division the divisor is (x – number).

A **solved example** is described below:

$$P(x) = 2x^3 + 3x^2 - 4$$
$$Q(x) = x + 1.$$

We want to divide P(x) by Q(x) using Ruffini's rule. The main problem is that Q(x) is not a binomial of the form x – number, but rather x + number. We must rewrite Q(x) this way:

$$Q(x) = x + 1 = x - (-1).$$

Now we apply the algorithm:

1st step: Write down the coefficients and r. Note that, as P(x) did not contain a coefficient for x, we have written 0:

```
     |    2     3     0     -4
     |
 -1  |
-----|----------------------------
     |
```

2nd step: Pass the first coefficient down:

```
     |    2     3     0     -4
     |
 -1  |
-----|----------------------------
     |    2
```

3rd step: Multiply the last obtained value by r:

```
     |    2     3     0     -4
     |
 -1  |         -2
-----|----------------------------
     |    2
```

4th step: Add the values:

```
     |    2     3     0     -4
     |
 -1  |         -2
-----|----------------------------
     |    2     1
```

5 th step: Repeat steps 3 and 4 until we've finished:

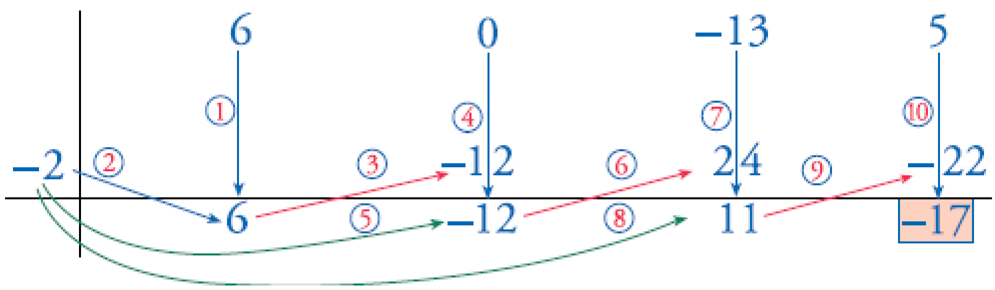
```
      |    2     3     0    -4
  -1  |         -2    -1     1
----- |---------------------------
      |    2     1    -1    -3
    {result coefficients}{remainder}
```

Remainder

Quotient

So, if the original number = divisor × quotient + remainder, then **P(x) = Q(x) R(x) + s**, where **R(x) = $2x^2 + x - 1$** and **s = −3**.

<u>Solved example 1</u>: Divide **$6x^3 - 13x + 5$** by **x + 2**



Therefore:

	6	0	−13	5
−2		−12	24	−22
	6	−12	11	$\boxed{-17}$

Remainder

Quotient

Quotient: **C(x) = $6x^2 - 12x + 11$**.

Remainder: **R = − 17**.

Example 3: Divide $P(x) = 2x^3 - 13x + 8$ into $Q(x) = x + 3$ using Ruffini's rule.

Example 4: Divide $P(x) = x^4 - 6x^3 + 9x + 10$ by $Q(x) = x - 3$ using Ruffini's rule.

Example 5: Divide $P(x) = x^5 - 4x^3 + 7x + 12$ by $Q(x) = x + 1$ using Ruffini's rule.

7. THE REMAINDER THEOREM.

The remainder in a division like P(x) : (x - a), where **a** is any number, is the numeric value of the polynomial for x = a, it is P(a).

Solved example 1: Without doing the division, what is the remainder of the division
$\mathbf{P(x) = x^3 - 7x + 15}$ into $\mathbf{x + 3}$?

Remainder = $\mathbf{R = P(-3) = (-3)^3 - 7 \cdot (-3) + 15 = -27 + 21 + 15 = 36 - 27 = 9}$

Solved example 1: Without doing the division, work out the remainder of the division
$\mathbf{P(x) = x^3 - 6x^2 + 5}$ into $\mathbf{x - 2}$.

Solved example 2: Without doing the division, what is the value of the remainder of the division $\mathbf{P(x) = x^4 + 3x^3 - 5x - 7}$ into $\mathbf{x + 3}$?

Example 3: Calculate the value of k, if the remainder of the division $\mathbf{(x^3 + kx - 6) : (x - 2)}$ is 5.

8. POLYNOMIAL ROOTS. (Raíces de polinomios)

The **root** of a polynomial $P(x)$ is a number a such that $P(a)=0$.

The fundamental theorem of algebra states that a polynomial $P(x)$ of degree **n** has **n** roots, some of which may be degenerate. For example, the roots of the polynomial

$$x^3 - 2x^2 - x + 2 = (x - 2)(x - 1)(x + 1)$$

are -1, 1, and 2. Finding roots of a polynomial is therefore equivalent to polynomial factorisation into factors of degree 1.

Example 1: Is 3 a root of the polynomial $P(x) = x^3 + x^2 - 9x - 9$? And -3 ?

Example 2: Calculate the roots of polynomial $P(x) = 2x^2 - 8x + 6$. Also, do the polynomial factorisation.

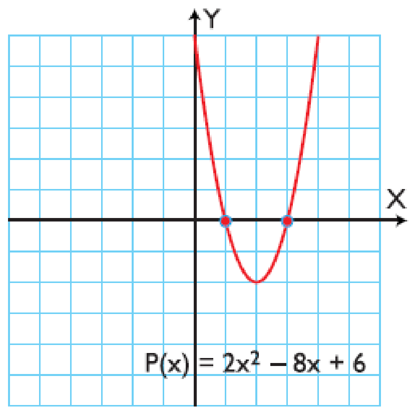
Example 3: Calculate the roots of polynomial $P(x) = x^2 + 9$. Also, do the polynomial factorisation.

Example 4. Do the polynomial factorisation of $P(x) = x^3 + 2x^2 - 5x - 6$.

Example 5: This example shows the graphic interpretation of roots of polynomials.

The roots of the polynomial are the points of intersection of its curve with the X axis:

Example 6: Identify the roots of the polynomial below:



PRACTISE MORE EXERCISES

Exercise 44. Divide $P(x) = 6x^6 - 13x^5 - 20x^3 + 50x^2 - 4$ into $Q(x) = 2x^3 - 3x^2 + 1$.

Exercise 45. Divide using Ruffini's rule $P(x) = x^4 - 6x^2 + 4x + 5$ into $Q(x) = x + 2$.

Exercise 46. Without doing the division, calculate the remainder of the division $P(x) = x^3 - 5x^2 + 7$ into $x - 3$.

Exercise 47. Without doing the division, calculate the remainder of the division $P(x) = x^4 - 2x^3 + 7x - 3$ into $x + 2$.

Exercise 48. Is 2 a root of polynomial $P(x) = x^3 + 2x^2 - x - 2$? And -2?

Exercise 49. Is the polynomial $P(x) = x^4 - 6x^3 + 8x^2 + 6x - 9$ divisible by $x - 3$?

Exercise 50. Calculate the value of k so that the reminder is 7, in the division $(x^4 + kx^2 - 5x + 6) : (x + 1)$.

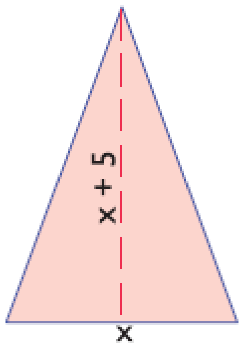
Exercise 51. Do the polynomial factorisation of the following polynomials:

a) $24x^3 - 18x^2$ b) $2x^3 + 12x^2 + 18x$

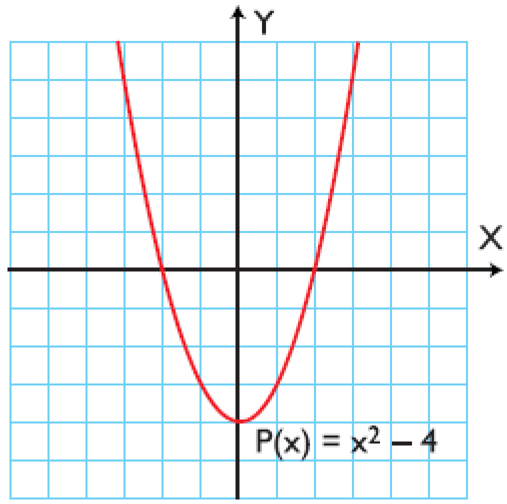
c) $9x^2 - 4$ d) $5x^4 - 10x^3 + 5x^2$

Exercise 52. Calculate the value of k so that $P(x) = x^3 + 5x^2 + kx - 8$ is divisible by $x + 2$.

Exercise 53. Calculate the polynomial which is the area of this triangle:



Exercise 54. Look at the graphical representation of the polynomial $P(x) = x^2 - 4$ and identify its roots.



Exercise 55. Calculate the polynomial which is the area of the rhombus below: