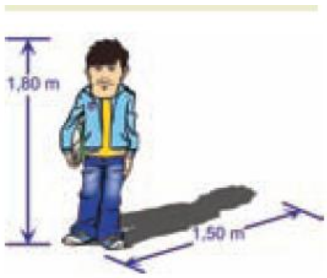
Unit 4. PROPORTIONS.

1. RATIO AND PROPORTION. (Razón y Proporción).

A ratio is a division of two comparable magnitudes.

Example:



Halla la razón entre la estatura de Diego y la longitud de su sombra. ¿Qué indica la razón?

$$\frac{\text{Estatura de Diego}}{\text{Longitud de su sombra}} = \frac{1{,}8}{1{,}5} = 1{,}2$$

La estatura de Diego es 1,2 veces la longitud de su sombra.

A proportion is the name that we give to a statement which says that two ratios are equal. It can be written in two ways:

- two equal fractions, $\frac{a}{b} = \frac{c}{d}$

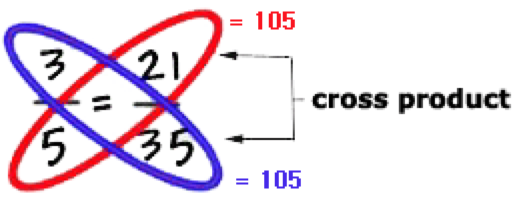
or,

- using a colon, **a:b = c:d**

When two ratios are equal, then the cross products of the ratios are equal.

That is, for the proportion, **a:b = c:d , a × d = b × c**

$$\frac{3}{5} = \frac{21}{35}$$



In problems involving proportions, we can use cross products to test whether two ratios are equal and form a proportion. To find the cross products of a proportion, we multiply the outer terms, called the extremes, and the middle terms, called the means.

The following proportion is read as "twenty is to twenty-five as four is to five."

$$\frac{20}{25} = \frac{4}{5}$$

Ejemplo

$$\frac{13,5}{5} = \frac{5,4}{2} = 2,7\ €$$

En las dos garrafas se obtiene el mismo precio por litro, que es la constante de proporcionalidad.

EXAMPLE:

Find whether each of the following statements is a proportion:

a) $\frac{2}{3} = \frac{6}{9}$ Using cross products to verify: 2 x 9 = 3 x 6.

So, it is a proportion.

b) $\frac{4}{3} = \frac{20}{18}$ Using cross products to verify: $4 \cdot 18 \neq 3 \cdot 20$.

So, it is not a proportion.

EXAMPLE: What value of "n" will make this a proportion? $\frac{6}{15} = \frac{n}{25}$

EXERCISES: Problem solving

1. I eat 7 cakes every week.
So every 2 weeks I eat ----------- cakes
and every 3 weeks I eat ----------- cakes.

2. For every 2 bags of crisps you buy you get 1 sticker,
For every 6 bags of crisps you buy you get ------- stickers.
To get 4 stickers you must buy ---------- of crisps.

3. Sarah uses 3 tomatoes for every ½ litre of sauce.
How many tomatoes does she need to for 1 litre of sauce?
How many tomatoes does she need to for 2 litres of sauce?

6. A mother seal is fed 5 fish for every 2 fish for its baby.
Alice fed the mother seal 15 fish.
How many fish did its baby get?

7. A mother seal is fed 5 fish for every 2 fish for its baby.
Alice fed the baby seal 8 fish.
How many fish did its mother get?

8. The juice of 5 oranges is used to make one jug of orangeade.

a. How many oranges will you need to make 2 jugs of orangeade?
What operation do you need to use?

b. How many jugs could you make using 36 oranges?
What operation do you need to use?

2. DIRECT PROPORTION.

Two variables are in direct proportion to one another if there is always the same ratio between their elements.

Values that are in direct proportion are often displayed in a table.

x	2	4	10
y	8	16	40

Notice that if x is doubled, then y is doubled too. All the values fit the equation **$y=4x$**.

The formal definition of a direct proportion is:

> Two quantities, A and B, are in direct proportion if by whatever factor A changes, B changes by the same factor.

Here is a shorthand way to say that the quantities A and B are directly proportional:

$$A \alpha B$$

The Greek letter between the A and the B is called alpha. It is here written in lower case. In this context it is shorthand for the phrase "is directly proportional to." So, the above statement reads "A is directly proportional to B."

How can we determine if two variables are directly proportional?

Think about these tables:

y	1	2	3	4	5	6
x	2	4	6	8	10	12
y/x	0.5	0.5	0.5	0.5	0.5	0.5

y	1	2	3	4	5	6
x	3	5	7	9	11	13
y/x	1/3	2/5	3/7	4/9	5/11	6/13

TO FIND MORE PRACTICAL EXERCISES ABOUT DIRECT PROPORTIONS, PLEASE VISIT THIS WEBPAGE.

http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/proporc_p.html

EXERCISE 1. Try the next interactive activity to understand ratios.

http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/flash1.shtml

EXERCISE 2. Do the following test to check what you have learnt about ratios and direct proportion

Test 1: Very easy

http://www.bbc.co.uk/apps/ifl/skillswise/mod_quizzes/numbers/wholenumbers/ratioandproportion/ratio/quizengine?quiz=simplequiz1;templateStyle=simplequiz;pagerType=pages;pagerData=1

Test 2: A bit harder

http://www.bbc.co.uk/apps/ifl/skillswise/mod_quizzes/numbers/wholenumbers/ratioandproportion/ratio/quizengine?quiz=simplequiz2;templateStyle=simplequiz;pagerType=pages;pagerData=1

Test 3: Only for confident students

http://www.bbc.co.uk/apps/ifl/skillswise/mod_quizzes/numbers/wholenumbers/ratioandproportion/ratio/quizengine?quiz=simplequiz3;templateStyle=simplequiz;pagerType=pages;pagerData=1

EXERCISE 3. Related and proportional quantities

If an egg costs 5 pence, 2 will cost 10 pence, and 3 will cost 15 pence.

In the table below we can see that the upper series of numbers have been multiplied by 5 to calculate the lower series:

Y	1	2	3	4
X	5	10	15	20

Two series of numbers are proportional when there is a multiplying factor or a dividing factor that allows us to create a series from the other.

However, there are related quantities that are not proportional. For example: a boy's age and his weight. If a six-year-old boy weighs 30 kilos, a twelve-year-old boy will not necessarily weigh 60 kilos. (6 x 2 years old, 30 x 2 Kg). At the age of 24 he will not necessarily weigh 120 kilos. (6 x 4 years old, 30 x 4 Kg).

Answer these questions saying if they are related or proporcional quantities:

	Related	*Proportional*
1. The amount of rice I sell and the money they pay me are...		

2. The age of a girl and its height are...		
3. The fertilizer we add to a field and the crop we obtain are...		
4. The number of carpenters and the number of chairs they make are...		

EXERCISE 4. Direct and inverse relationships.

The relationships we have studied were direct, that is, when one quantity gets bigger, the other also gets bigger.

Now we are going to see inverse relationships, that is, as one quantity gets bigger, the other gets smaller.

Exemple: an old man's age and the strength he has. As age increases, his strength decreases. It is an inverse relationship.

Answer saying if they are direct or inverse relationships:

	Direct relationship	*Inverse relationship*
1. The speed of a train and the time it takes to travel between two cities is...		
2. The number of workmen in a factory and the number of pieces of furniture they make is...		
3. The power of a car and its speed is...		
4. The time it takes to build a road and the number of workmen working is...		
5. The number of copules that get married and the number of children that are born is...		
6. The size of a hole in a barrel and the time it takes for it to empty is...		

EXERCISE 5. Direcly proportional quantities and inversely proportional quantities.

When the relationship between two quantities is exact, it is called a proportion.

Two quantities are directly proporcional if when we multiply the amount of one, the amount of the other gets multiplied by the same factor.

Example: A packet of tobacco costs 2 euros; 3 packets cost 6 euros (2 x 3).

They are inversely proportional if when we multiply the amount of one, the amount of the other ges divided by the same factor.

Example: If a reaper takes 21 hours to reap a field, 7 reapers will take 3 hours. (21 :7).

Answer saying whether these proportions are direct or inverse:

	Directly proportional	*Inversely proporcional*
1. The number of woodcutters and the number of trees they can fell is...		
2. The speed of a plane and the time it takes to fly to its destination is...		
3. The amount of cigarettes I smoke and the amount of money I spend is...		
4. The number of notebooks I buy and the money I have to pay is...		
5. If you have 12 euros to buy books, the number of books that you can buy and their price is...		
6. The number of painters and the time they take to paint a house is...		

HOW TO SOLVE A PROBLEM WITH A DIRECT PROPORTION (direct rule of three) (=regla de tres directa)

The direct rule of three is used to calculate the fourth quantity in a direct proportion, when we know the other three quantities.

La **regla de tres directa** es un procedimiento que tiene por objeto hallar el cuarto proporcional. Se plantea de la siguiente forma:

a) Se colocan los datos y se determina si la proporcionalidad es directa:

Magnitud A (unidad)	(D)	Magnitud B (unidad)
a	⟶	c
b	⟶	x

b) Se forma la proporción y se calcula el cuarto proporcional:

$$\frac{a}{b} = \frac{c}{x} \Rightarrow x = \frac{b \cdot c}{a}$$

EXERCISE 1. If 10 calculators cost £120, how much will 8 calculators cost?

EXERCISE 2. Practice Questions

Work out the answer to each of these questions:

(a) If 5 bags of sweets contain 90 sweets in total, calculate how many sweets seven bags will contain.

(b) If 25 CDs cost £5.50, calculate the cost of 11 CDs.

EXERCISE 3. Work out the answers to the questions below and fill in the boxes.
If 5 tickets for a play cost £40, calculate the cost of:

(a) 6 tickets ……………………

(b) 9 tickets ………..

(c) 20 tickets ……………

EXERCISE 4. To make 3 glasses of orange squash you need 600ml of water.
Work out how much water you need to make:

(a) 5 glasses of squash …………… ml

(b) 7 glasses of squash……………… ml Check

(c) 1 glass of squash………………… ml Check

EXERCISE 5. If 10 litres of petrol cost £8.20, calculate the cost of:

(a) 4 litres…………………………

(b) 12 litres……………………….

(c) 30 litres………………………

EXERCISE 6. A baker uses 1,800 grams of flour to make 3 loaves of bread.
How much flour will he need use if he wants to make:

(a) 1 loaf……………………grams

(b) 7 loaves……………………grams

(c) 24 loaves……………………grams

EXERCISE 7. Ben buys 21 football stickers for 84p. Calculate the cost of:

(a) 7 stickers ……………………………………………pence

(b) 12 stickers……………………………………. pence

(c) 50 stickers……………………………………. pence

EXERCISE 8. 16 teams, each with the same number of people, enter a quiz.
At the semi-final stage there are 12 people left in the competition.

How many people entered the competition? …………people

EXERCISE 9. Three identical coaches can carry a total of 162 passengers.

How many passengers can be carried on seven of these coaches? ………….people

3. INVERSE PROPORTION.

In an **Inverse proportion**, when one value increases the other value decreases by the same factor.

Example:

HOW TO SOLVE A PROBLEM WITH AN INVERSE PROPORTION: (inverse rule of three) (regla de tres inversa)

The inverse rule of three is used to calculate the fourth quantity in an inverse proportion, when we know the other three quantities.

Se aplica el siguiente procedimiento:

a) Se colocan los datos y se determina si la proporcionalidad es inversa:

Magnitud A (unidad)	(I)	Magnitud B (unidad)
a	$\longrightarrow$	c
b	$\longrightarrow$	x

b) Se forma la proporción en la que la razón de las cantidades de la magnitud A aparece invertida:

$$\frac{b}{a} = \frac{c}{x} \Rightarrow x = \frac{a \cdot c}{b}$$

Ejemplo

Para almacenar una colección de cómics hemos utilizado 60 carpetas con 4 cómics cada una. Si se quieren almacenar 5 cómics en cada carpeta, ¿cuántas se necesitarán?

a) Se colocan los datos: b) Se forma la proporción:

Nº comics	(I)	Nº Carpetas
4	$\longrightarrow$	60
5	$\longrightarrow$	x

$$\Rightarrow \frac{5}{4} = \frac{60}{x} \Rightarrow x = \frac{4 \cdot 60}{5} = 48 \text{ carpetas}$$

Invertido para formar proporción

EXERCISE 1. At the rate of 28 lines per page, a book has 300 pages. If the book has to contain only 280 pages, how many lines should a page contain?

EXERCISE 2. A farmer has enough grain to feed 50 cattle for 10 days. He sells 10 cattle. For how many days will the grain last now?

EXERCISE 3. In an army camp, there is food for 8 weeks for 1,200 people. After 3 weeks, 300 more soldiers join the camp. How many weeks will the food last?

EXERCISE 4. It is known that current (A) in an electric circuit is inversely proportional to the resistance (R) in the circuit. When the resistance is 3 ohms, the current is 2 amperes. Find the resistance if the current is 5 amperes; and find the current when the resistance is 6 ohms.

EXERCISE 5. Some people working at the rate of 6 hours a day can complete the work in 19 1/2 days. As they have received another contract, they want to finish this work early. Now they start working 6 1/2 hours a day. How many days will they take to finish this work?

EXERCISE 6. If y varies inversely as x; and x=6 when y=-3, find y when x=-9.

EXERCISE 7. 35 trucks are needed to transport all the grain in a warehouse at the rate of 80 bags a truck. But the truck drivers objected saying it was too heavy and carried only 70 bags a truck. How many more trucks are needed to transport the grain?

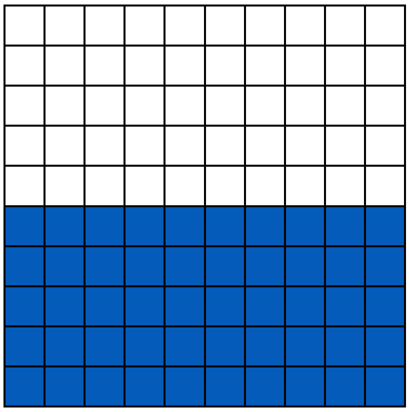
4. PERCENTAGES.

4.1. What does % mean?

You probably already know a bit about percentages. Shops use percentages in sales. Banks use them for loan rates. Weather forecasts use them to tell us the chances of rain. But what does **percentage** mean?

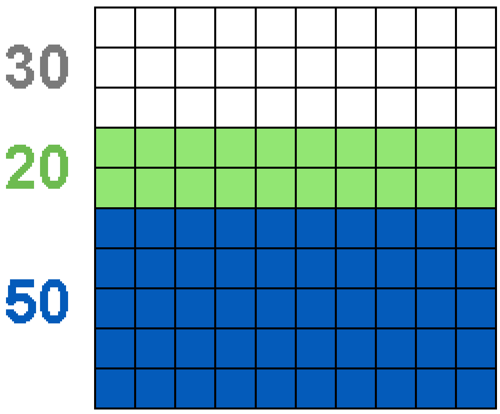
'per cent' means 'out of every 100'

Look at this square. It has been divided into 100 equal parts. 50 parts are shaded blue.



We say that **fifty per cent** of the square is shaded blue.

You can write this using a percentage symbol as 50%



Now 20 parts have been coloured green. 20 out of the 100 is 20%, so 20% of the square is green.

There are 30 parts not shaded. 30 out of 100 is 30%, so 30% of the square is white.

What happens if you add up the percentages for the blue, green and white parts?

50% + 20% + 30% = 100%

So the whole square is equal to 100%

100% is '100 out of 100'. It means full marks in a test, or 100% fitness means full fitness. If you give 100% you give everything.

4.2. Fractions and percentages

Skirts are on sale for 50% off the original price of £20. How much are they now?

If you said £10 well done! It is quite easy to find the answer if you can remember that finding 50% is the same as finding a half, or dividing by two.

Here are the fractions for some other percentages:

percentage	fraction
10%	$\frac{1}{10}$
20%	$\frac{1}{5}$
25%	$\frac{1}{4}$
50%	$\frac{1}{2}$
75%	$\frac{3}{4}$

Examples:

20% is the same as $\frac{1}{5}$, so divide by 5 to find the saving.

10% is the same as $\frac{1}{10}$, so divide by 10 to find the saving.

Example

To pass a national exam you need to get 75%. There are 40 questions altogether. How many do you need to get right?

75% is the same as $\frac{3}{4}$, so find $\frac{1}{4}$ then multiply by 3 to find $\frac{3}{4}$.

$\frac{1}{4}$ of 40 = 40 ÷ 4 = 10

$\frac{3}{4}$ of 40 = 3 x 10 = 30

So you need to get **30** questions right to pass the exam.

4.3. Comparing sizes

It is quite easy to compare percentages of one particular thing. For example, if your boss offered you the choice of a 10% pay rise or a 20% pay rise, which would you choose?

If you went for 20%, well done!

20% is the better pay rise, because it is a bigger proportion of your wage than 10%.

You have to be a bit more careful if you are finding, or comparing, percentages of different things.

Example

What happens if everyone in a company gets an increase of 50%? They would not all get the same pay rise, because the size of 50% increase depends on the original wage.

Here are some examples:

wage	50%	new wage
£100	£50	£150
£150	£75	£225
£200	£100	£300

Example

Which is the best bargain? You must work out which is the best value.

50% off £16 is half price. So it costs £8

25% off is a quarter off. So the £10 item now costs £7.50

Can you see that even though the second one had less of a percentage saving it was still the best buy?

To know the value of a percentage you must know what it is a percentage of.

Here there are **two** activities to see what percentages look like. In the second one you can compare percentages and fractions.

Percentages of something.

http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/percentages/introduction/flash2.shtml

See percentages and fractions side by side.

http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/percentages/introduction/flash3.shtml

Do the following interactive exercises to learn how to calculate percentages.

Exercise 1.
http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/txc/txc_p.html

Exercise 2

http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/txcc/txcc_p.html

Exercise 3.

Calculate 30% of 1000, and 20% of 300.

Exercise 4.

Read the following sheets:

Sheet 1
(http://www.isftic.mepsyd.es/w3/recursos/primaria/matematicas/porcentajes/contenidos/unidad2.html),

sheet 2
(http://www.isftic.mepsyd.es/w3/recursos/primaria/matematicas/porcentajes/contenidos/unidad3.html) and

watch this **video** (http://www.mathplayground.com/howto_perfracdec.html)

In the next **website**
(http://www.isftic.mepsyd.es/w3/recursos/primaria/matematicas/porcentajes/menuu2.html)

do the exercises "Cálculo mental" and "Test".

In the next **website**
(http://www.isftic.mepsyd.es/w3/recursos/primaria/matematicas/porcentajes/menuu3.html)

do the exercises "Tú mandas", "Cuadrolos", "% Coloreado" and "Test".

Do the following tests to check what you have learnt about percentages.

Test 1: Very easy

Solutions:

Test 2: A bit harder

Solutions:

Test 3: Only for confident students

Solutions:

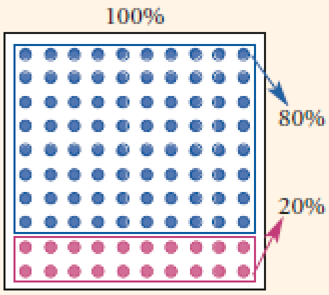
Test 4: Another test.

Solutions:

REMEMBER THAT:

Disminución porcentual

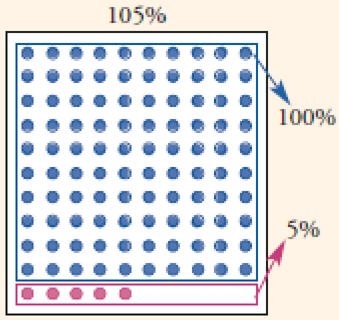
Disminuir una cantidad en un 20% es lo mismo que calcular el 100% – 20% = 80% de la cantidad.



$$80\% = \frac{80}{100} = 0{,}8$$

Aumento porcentual

Aumentar una cantidad en un 5% es lo mismo que calcular el 100% + 5% = 105% de la cantidad.



$$105\% = \frac{105}{100} = 1{,}05$$

4.4. LINKED PERCENTAGES.

To calculate linked percentage increases and decreases, we calculate the total variation index multiplying the variation indexes in each step. In Spanish:

Para calcular **aumentos y disminuciones porcentuales encadenados** se calcula el índice de variación total multiplicando los índices de variación de cada paso.

Ejemplo

Un componente informático que costaba 100 € el año pasado ha aumentado su precio un 10%. Al comprarlo este año, nos rebajan un 10%. ¿Qué precio pagamos por él?

Índice de variación del aumento porcentual: 1,1

Índice de variación de la disminución porcentual: 0,9

Índice total: $1{,}1 \cdot 0{,}9 = 0{,}99$

Se paga un 99% del precio, que es $100 \cdot 0{,}99 = 99$ €

Exercise 1. Page 79, exercise 23. Copy the instructions and solve it.

Exercise 2. Page 79, exercise 24. Copy the instructions and solve it.

Exercise 4. Page 79, exercise 25. Copy the instructions and solve it.

5. COMPOUND PROPORTION.

A proportion is compound if we have more than two proportional magnitudes.

Para resolver problemas por la regla de tres compuesta se sigue el procedimiento:

a) Se identifican las magnitudes que intervienen y sus unidades.

Se colocan los datos poniendo en último lugar la incógnita.

Se determina la proporcionalidad que hay entre cada magnitud y la magnitud de la incógnita.

¿Proporcionalidad directa o inversa?

¿Proporcionalidad directa o inversa?

Magnitud A (unidad)		**Magnitud B (unidad)**		**Magnitud C (unidad)**
a	$\longrightarrow$	c	$\longrightarrow$	e
b	$\longrightarrow$	d	$\longrightarrow$	x

b) Se plantea la proporción, con la razón directa o inversa, según sean las magnitudes directa o inversamente proporcionales, y se resuelve.

Ejemplo

Dos obreros canalizan 100 m de tubería para agua durante 10 días. ¿Cuántos días tardarán en canalizar 350 m de tubería 5 obreros?

(I) (D)

a) Nº de obreros		Longitud (m)		Tiempo (días)
2	$\longrightarrow$	100	$\longrightarrow$	10
5	$\longrightarrow$	350	$\longrightarrow$	x

b) $\frac{5}{2}\cdot\frac{100}{350}=\frac{10}{x}\Rightarrow\frac{50\cancel{0}}{70\cancel{0}}=\frac{10}{x}\Rightarrow x=\frac{70}{5}=14$ días

Razón invertida para formar proporción

Solved example: A farmer needs 600 kg of fodder to feed 40 cows for 8 days. How many days could he feed 20 cows with 1,500 kg of fodder?

For 40 cows:

Weight	(Direct Proportion)	Days
600kg		8 days
1,500 kg ………..		… x days

$\Rightarrow$ x = $\frac{1500\cdot 8}{600}=20$ days for 40 cows.

With 1,500 kg for 20 cows:

Cows	(Inverse Proportion)	Days
40 cows ……………. 20 days		
20 cows……………… y days		

$\Rightarrow$ y= $\frac{40\cdot 20}{20}=40$ days for 20 cows with 1,500 kg

Solution: 40 days for 20 cows with 1,500 kg.

EXERCISE 1. Page 77, exercise 12. Copy the instructions.

EXERCISE 2. Page 77, exercise 13. Copy the instructions.

EXERCISE 3. Page 77, exercise 14. Copy the instructions.

6. DIRECTLY PROPORTIONAL DISTRIBUTIONS.
(=Repartos directamente proporcionales)

Para repartir una cantidad **N** en partes que sean directamente proporcionales a otras cantidades conocidas **a, b, c…,** se sigue el procedimiento:

a) Se calcula la parte de **N** que le corresponde a cada unidad del total de las cantidades conocidas **a, b, c…,** es decir:

$$k = \frac{N}{a + b + c}$$

b) Con el valor de la unidad, **k,** se calculan los valores de las partes deseadas.

Solved example:

Reparte 3 000 € en partes proporcionales a 4, 7 y 9

a) Cálculo del dinero que corresponde a una unidad: $\frac{3\,000}{4+7+9} = 150$ €

b) Cálculo de las partes:

$x = 150 \cdot 4 = 600$ €

$y = 150 \cdot 7 = 1\,050$ €

$z = 150 \cdot 9 = 1\,350$ €

Exercise 1. Page 79, exercise 19. Copy the instructions and solve it.

Exercise 2. Three friends organise a club to play football pools and they invest €23, €34 and €41 respectively. If they win the pools and earn € 120,540, what is the corresponding part for every one if the distribution is done directly proportionally?

7. INVERSELY PROPORTION DISTRIBUTIONS.
(=Repartos inversamente proporcionales)

Para repartir una cantidad **N** en partes que sean inversamente proporcionales a otras cantidades conocidas **a, b, c…**, se hace un reparto directamente proporcional a las inversas **1/a, 1/b, 1/c…** Para ello:

a) Se calcula primero el inverso de **a, b, c…**, y se reducen a común denominador.

b) Se hace el reparto directamente proporcional a los numeradores.

Ejemplo

Reparte 4 200 € en partes inversamente proporcionales a 3, 5 y 6

a) Se calculan los inversos y se reducen a común denominador:

$$\text{m.c.m.}(3, 5, 6) = 30 \Rightarrow \frac{1}{3} = \frac{10}{30}, \ \frac{1}{5} = \frac{6}{30}, \ \frac{1}{6} = \frac{5}{30}$$

b) Se hace el reparto directamente proporcional a **10**, **6** y **5**

A cada unidad le corresponden: $\frac{4\,200}{10 + 6 + 5} = 200$ €

$x = 200 \cdot 10 = 2\,000$ €

$y = 200 \cdot 6 = 1\,200$ €

$z = 200 \cdot 5 = 1\,000$ €

Exercise 1. Page 79, exercise 20. Copy the instructions and solve it.

Exercise 2. A father shares out €13,440 among his three children in inverse proportion to their ages (the youngest one gets the most money); they are 5, 12 and 15 years old. Calculate the corresponding part for every one.

MATHS IS FUN