UNIT 2. POWERS AND ROOTS

1. POWERS.

1.1 DEFINITION.

Complete the following table:

1	2	3	4	5	6	7	8	9	10
1	4					49			

When you multiply two or more numbers, each number is called a **factor of the product**. When the same factor is repeated, you can use an **exponent** to simplify your writing. An exponent tells you how many times a number, called the **base**, is used as a factor.

A power is a number that is expressed using exponents.

exponent

base → $a^n = \overbrace{a \cdot a \cdot \ldots \cdot a}^{n}$

In English: base → ……………………………… Exponente →…………………………

Other examples:

- 45^5 = 45 elevado a la quinta potencia = forty-five to the fifth power / forty-five to the power of five
- 15^{21} – fifteen to the twenty-first power / fifteen to the power of twenty-one

Exercise 1. Calculate:

a) $(-2)^3 =$

f) $2^3 =$

b) $(-3)^3 =$

g) $(-1)^4 =$

c) $(-5)^4 =$

h) $(-5)^3 =$

d) $(-10)^3 =$

i) $(-10)^6 =$

e) $(7)^3 =$

j) $(-7)^3 =$

REVISE HOW TO USE THE CALCULATOR.

There are some very useful keys to do powers:

$\boxed{x^2}$ squared (elevar al cuadrado) E.g. 17^2 $\boxed{17}\ \boxed{x^2}\ \boxed{=}\ \boxed{289}$

$\boxed{x^3}$ cubed (elevar al cubo) E.g. 14^3 $\boxed{14}\ \boxed{x^3}\ \boxed{=}\ \boxed{2744}$

$\boxed{\wedge}$ or $\boxed{x^y}$ x raised to the yth power . E.g. $\boxed{5}\ \boxed{\wedge}\ \boxed{6}\ \boxed{=}\ \boxed{15625}$

$\boxed{EXP}$ 10 raised to the xth power E.g. $\boxed{EXP}\ \boxed{5}\ \boxed{=}\ \boxed{100000}$

Exercise: Calculate with the calculator:

a) $(-6)^2 =$ b) $5^3 =$ c) $(2)^{20} =$

d) $(10)^8 =$ e) $(-6)^{12} =$

1.2. PROPERTIES OF POWERS.

Here are the properties of powers. Pay attention to the last one (section vii, powers with negative exponent) because it is something new for you:

i) Multiplication of powers with the same base: $a^n \cdot a^p = a^{n+p}$

E.g.: $7^2 \cdot 7^3 = 7^5$ $7^2 \cdot 7^3 = \overbrace{7 \cdot 7}^{2} \cdot \overbrace{7 \cdot 7 \cdot 7}^{3} = 7^5$

ii) Division of powers with the same base : $\dfrac{a^n}{a^p} = a^{n-p}$

E.g.: $\dfrac{6^5}{6^3} = 6^2$ $\dfrac{6^5}{6^3} = \dfrac{\cancel{6} \cdot \cancel{6} \cdot \cancel{6} \cdot 6 \cdot 6}{\cancel{6} \cdot \cancel{6} \cdot \cancel{6}} = 6 \cdot 6 = 6^2$

E.g.: $3^5 : 3^4 = 3^1 = 3$

iii) Power of a power: $(a^n)^p = a^{n \cdot p}$

E.g. $(3^5)^2 = 3^{10}$

Checking: $(3^5)^2 = 3^5 \cdot 3^5 = (3 \cdot 3 \cdot 3 \cdot 33) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^{10}$

iv) Power of a multiplication: $(a \cdot b)^n = a^n \cdot b^n$

E.g. $(3 \cdot 5)^3 = 3^3 \cdot 5^3$

v) Power of a division: $(a : b)^n = a^n : b^n$

E.g.: $(3 : 5)^3 = 3^3 : 5^3 = \dfrac{27}{125}$

vi) Remember this: $a^0 = 1$**, so any number powered 0 is equal to 1.**

Examples: $5^0 = 1$, $2^0 = 1$, $(0.5)^0 = 1$, $(-5)^0 = 1$

Checking: Applying the rule for division of powers with the same base: $a^n : a^p = a^{n-p}$, we can see that $a^n : a^n = a^{n-n} = a^0$. Any number divided by itself is equal to 1, so any power (a^n) divided by itself ($a^n : a^n = a^0$) is equal to 1.

vii) Powers with a negative exponent. $x^{-n} = \dfrac{1}{x^n}$

Example: $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ **; here's why:**

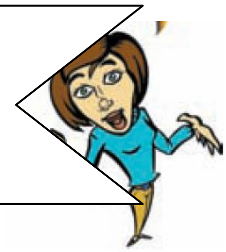
$$\begin{cases} \dfrac{4^3}{4^5} = 4^{3-5} = 4^{-2} \\ \dfrac{4^3}{4^5} = \dfrac{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4} = \dfrac{1}{4^2} \end{cases}$$

SUMMING UP:

DEFINITION		Power = $a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n\ \text{times}}$ a= base n= exponent
RULES OF POWERS	SAME BASE	➢ When multiplying => Add the exponents $2^3 \times 2^4 = 2^{3+4} = 2^7$ ➢ When dividing => Subtract the exponents $2^5 : 2^3 = 2^{5-3} = 2^2$
	SAME EXPONENT	➢ When multiplying => Multiply the base $2^3 \times 5^3 = (2 \cdot 5)^3 = 10^3$ ➢ When dividing => Divide the base $8^5 : 2^5 = (8 : 2)^5 = 4^5$
	POWER OF A POWER	Multiply the exponents Ex. $(2^3)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^{2 \cdot 3} = 2^6$ $(2^2)^4 = 2^{4 \cdot 2} = 2^8$

To revise these rules, you can visit this video on the Internet:

http://www.math-videos-online.com/exponents-rules.htmll



Exercise 1: The most usual errors with powers are in the following examples, find them:

a) $2^3 = 6$?

b) $3^0 = 0$?

c) $-2^2 = -4$?

d) $(2+3)^2 = 2^2 + 3^2$?

e) $(3-1)^2 = 3^2 - 1^2$?

f) $3^{-2} = -3^2$?

Exercise 2: Write as a power:

a) $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 =$

b) $-3 \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) =$

Exercise 3: Calculate in your mind:

a) 2^0 ____ b) 2^1 ____ c) 2^2 ____ d) 2^3 ____ e) 2^4 ____ f) 2^5 ____

Exercise 4: Calculate in your mind:

a) 10^0 ____ b) 10^1 ____ c) 10^2 ____ d) 10^3 ____ e) 10^4 ____ f) 10^5 ____

Exercise 5: Calculate in your mind:

a) $(-3)^0$ _____ b) $(-3)^1$ _____ c) $(-3)^2$ _____ d) $(-3)^3$ _____ e) $(-3)^4$ _____

Exercise 6: Calculate in your mind:

a) $-2^3 =$ _____ b) $-3^3 =$ _____ c) $-2^4 =$ _____ d) $-3^4 =$ _____ e) $-10^2 =$ _____

Exercise 7: Use the properties of powers:

a) $5^3 \cdot 5^4 =$ b) $5^9 : 5^3 =$ c) $(5^3)^2 =$

d) $5^3 \cdot 7^3 =$ e) $5^4 : 7^4 =$

Exercise 8: Write as a power with an integer base:

a) $\frac{1}{5^3}$ b) $\frac{1}{16}$ c) $\frac{1}{3^2}$ d) $\frac{1}{81}$

....

To practise with exponents, you can visit this website:

http://www.mathsisfun.com/algebra/negative-exponents.html

2. ROOTS.

2.1. SQUARE ROOT.

Fill the table:

Number	1	2	3	4	5	6	7
Perfect Square (Cuadrado Perfecto)	1	4					

We usually write:

$\sqrt{4} = 2$ $\sqrt{1} = 1$ $\sqrt{9} = 3$

But this is not absolutely true, pay attention: $\sqrt{a} = b$ si $b^2 = a$

and then, look here:

$\sqrt{4} = \pm 2$ because $2^2 = 4$ and $(-2)^2 = 4$

$\sqrt{9} = \pm 3$ because $3^2 = 9$ and $(-3)^2 = 9$

$\sqrt{1} = \pm 1$ *because* $1^2 = 1$ *and* $(-1)^2 = 1$

$\sqrt{0} = 0$

$\sqrt{-9} = no\ existe = \nexists$

So, **a number can have two square roots, one or none.**
Write an example of each case:

- Two square roots: ....
- One square root: ....
- No square roots: ....

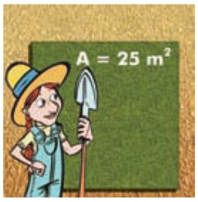
E.g.: How many roots has $\sqrt{4}$ got ? Two roots, they are 2 and -2, because $2^2 = 4$ and $(-2)^2 = 4$.

E.g.: How many roots has $\sqrt{-16}$ got?

E.g.: How many roots has $\sqrt{0}$ got ?

E.g.: How many roots has $\sqrt{81}$ got? ....

PROBLEM:



A gardener has a 25 m² square garden. How many metres long is the side?

LET'S APPROXIMATE SQUARE ROOTS:

a) $5 < \sqrt{27} < 6$ because $5^2 = 25$ and $6^2 = 36$.

b)............ $< \sqrt{10} <$ .... because ....

c)............ $< \sqrt{17} <$ .... because ....

d)............ $< \sqrt{70} <$ .... because ....

PROPERTIES OF SQUARE ROOTS.

i) $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$. Example: $\sqrt{12} \cdot \sqrt{3} = \sqrt{12 \cdot 3} = \sqrt{36} = \pm 6$

ii) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$. Example: $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = \pm 2$

N.B.: COMMON MISTAKES!

- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. Example: $\sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$ because $\sqrt{25} = 5 \neq 3 + 4$.
- $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$. Example: $\sqrt{25-9} \neq \sqrt{25} - \sqrt{9}$ because $\sqrt{16} = 4 \neq 5 - 3$.

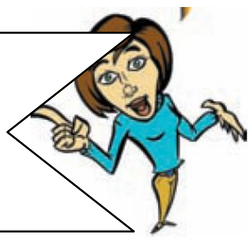
EXTRACTING THE FACTORS OF A ROOT:

Examples:

- $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{2^2 \cdot 3} = 2\sqrt{3}$
- $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$
- $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{3^2 \cdot 2} = 3\sqrt{2}$
- $\sqrt{62} =$ ....
- $\sqrt{75} =$ ....
- $\sqrt{200} =$ ....
- $\sqrt{20} =$ ....
- $\sqrt{45} =$ ....
- $\sqrt{48} =$ ....
- $\sqrt{2^2 \cdot 3^2 \cdot 5} =$ ....
- $\sqrt{7^2 \cdot 3^4} =$ ....
- $\sqrt{2^6 \cdot 5^4} =$ ....

To revise how to simplify roots, you can visit this video on the Internet:

http://www.math-videos-online.com/simplifying-square-roots.html



2.2. CUBE ROOT.

- $\sqrt[3]{8} = 2$ because $2^3 = 8$
- $\sqrt[3]{27} = 3$ because $3^3 = 27$
- $\sqrt[3]{1} = 1$ because $1^3 = 1$
- $\sqrt[3]{125} =$........ because ....
- $\sqrt[3]{0} =$........ because ....
- $\sqrt[3]{-8} =$........ because ....

LET'S APPROXIMATE CUBE ROOTS:

a) $1 < \sqrt[3]{4} < 2$ because $1^3 = 1$ and $2^3 = 8$.

b) .... $< \sqrt[3]{8} <$ .... because ....

c) .... $< \sqrt[3]{33} <$ .... because ....

d) .... $< \sqrt[3]{77} <$ .... because ....

PROPERTIES OF CUBE ROOTS.

i) $\sqrt[3]{a}\cdot\sqrt[3]{b}=\sqrt[3]{a\cdot b}$. Example: $\sqrt[3]{25}\cdot\sqrt[3]{5}=\sqrt[3]{125}=5$

ii) $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}=\sqrt[3]{\frac{a}{b}}$. Example: $\frac{\sqrt[3]{24}}{\sqrt[3]{3}}=\sqrt[3]{\frac{24}{3}}=\sqrt[3]{8}=2$

2.3. UMPTEENTH ROOT (RAÍZ n-ésima)

The umpteenth root of a number "a" is another number "b" so that $b^n = a$.
So:

$\sqrt[n]{a} = b$ if $b^n = a$

Example:

$\sqrt[4]{81}=\pm 3=\left\langle\begin{matrix}3\\-3\end{matrix}\right.$ because $\begin{cases}3^4=81\\(-3)^4=81\end{cases}$

2.4. EQUIVALENT ROOTS

Two roots are equivalent if they have got the same solutions. To get equivalent roots you can multiply or divide the index and exponent by the same number.

Example:

$$\sqrt[3]{5^2}=\sqrt[6]{5^4}=\sqrt[9]{5^6}=\sqrt[12]{5^8}=\ldots=2{,}92\ldots$$

Example: Simplifying a root.

$$\sqrt[18]{5^{12}}\underset{\uparrow}{=}\sqrt[18:6]{5^{12:6}}=\sqrt[3]{5^2}$$

M.C.D. (12, 18) = 6

2.5. PUTTING FACTORS INTO A ROOT.

To put a number into a root you have to raise it to the power of the index of the root.
Example:

$$2\sqrt[3]{5}=\sqrt[3]{2^3\cdot 5}=\sqrt[3]{8\cdot 5}=\sqrt[3]{40}$$

Other examples:

2.6. EXTRACTING THE FACTORS OF A ROOT:

Let's learn how to extract the factors of a root using some examples:
Example 1:

$$\sqrt[3]{32a^{17}b^{12}c^7}=\sqrt[3]{2^5a^{17}b^{12}c^7}=2a^5b^4c^2\sqrt[3]{2^2a^2c}$$

Example 2:

$$\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{2^3 \cdot 5} = 2\sqrt[3]{5}$$

Example 3:

$$\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = \sqrt[3]{5^3 \cdot 2} = 5\sqrt[3]{2}$$

Other examples:

- $\sqrt[3]{500} =$ ....
- $\sqrt[3]{54} =$ ....
- $\sqrt[3]{40} =$ ....
- $\sqrt[3]{1000} =$ ....
- $\sqrt[3]{135} =$ ....
- $\sqrt{81a^5bc^6} =$ ....
- $\sqrt[3]{128a^8b^2c^{15}} =$ ....
- $\sqrt{243a^8b^3c^7} =$ ....
- $\sqrt[3]{125a^9b^{17}c^{25}} =$ ....

2.7. ADDITION AND SUBTRACTION OF ROOTS

Two roots are similar (in Spanish: radicales semejantes) if they have got the same index and the same radicant. We can't add two roots if they aren't similar roots.
Let's study the following examples:

Example: $4\sqrt{50} - 7\sqrt{8} + 5\sqrt{18} =$

Example: $\sqrt{50} - \sqrt{32} + \sqrt{18}$

Example: $5\sqrt{98} - 3\sqrt{200} + 4\sqrt{8}$

2.8. PROPERTIES OF ROOTS.

FORMULA	EXAMPLE
$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$	$\sqrt[3]{5} \cdot \sqrt[3]{25} = \sqrt[3]{5 \cdot 25} = \sqrt[3]{125} = 5$
$\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a : b}$	$\sqrt[3]{32} \cdot \sqrt[3]{4} = \sqrt[3]{32 : 4} = \sqrt[3]{8} = 2$
$\left(\sqrt[n]{a}\right)^p = \sqrt[n]{a^p}$	$\left(\sqrt[3]{5}\right)^2 = \sqrt[3]{5^2}$
$\sqrt[n]{\sqrt[p]{a}} = \sqrt[n \cdot p]{a}$	$\sqrt[3]{\sqrt{5}} = \sqrt[6]{5}$

2.9. SUMMARY OF THE PROPERTIES OF POWERS AND ROOTS.

POWERS		ROOTS	
$a^n = a \cdot a \cdot \ldots^{n)} \ldots \cdot a$	$2^3 = 2 \cdot 2 \cdot 2 = 8$	$\sqrt{a} = b$ si $b^2 = a$	$\sqrt{25} = \pm 5$
$0^n = 0, 0 \neq 0$	$0^5 = 0$	$\sqrt[3]{a} = b$ si $b^3 = a$	$\sqrt[3]{8} = 2$
$1^n = 1$	$1^3 = 1$	$\sqrt[n]{a} = b$ si $b^n = a$	$\sqrt[5]{32} = 2$
$a^0 = 1, a \neq 0$	$5^0 = 1$	$\sqrt[n]{a^n} = \left(\sqrt[n]{a}\right)^n = a$	$\sqrt[5]{7^5} = \left(\sqrt[5]{7}\right)^5 = 7$
$a^1 = a$	$4^1 = 4$	$\sqrt[ns]{a^{ps}} = \sqrt[n]{a^p}$	$\sqrt[10]{7^6} = \sqrt[5 \cdot 2]{7^{3 \cdot 2}} = \sqrt[5]{7^3}$
$a^n \cdot a^p = a^{n+p}$	$5^3 \cdot 5^4 = 5^7$	$a\sqrt[n]{b} = \sqrt[n]{a^n \cdot b}$	$2\sqrt[3]{5} = \sqrt[3]{2^3 \cdot 5} = \sqrt[3]{40}$
$a^n : a^p = a^{n-p}$	$2^8 : 2^3 = 2^5$	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$	$\sqrt[5]{2} \cdot \sqrt[5]{3} = \sqrt[5]{6}$
$(a^n)^p = a^{n \cdot p}$	$(5^3)^2 = 5^6$	$\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a : b}$	$\sqrt[3]{2} : \sqrt[3]{5} = \sqrt[3]{2 : 5}$
$(a \cdot b)^n = a^n \cdot b^n$	$(2 \cdot 3)^5 = 2^5 \cdot 3^5$	$\left(\sqrt[n]{a}\right)^p = \sqrt[n]{a^p}$	$\left(\sqrt[3]{7}\right)^2 = \sqrt[3]{7^2}$
$(a : b)^n = a^n : b^n$	$(5 : 7)^3 = 5^3 : 7^3$	$\sqrt[n]{\sqrt[p]{a}} = \sqrt[n \cdot p]{a}$	$\sqrt[5]{\sqrt[3]{7}} = \sqrt[15]{7}$
$a^{-n} = \dfrac{1}{a^n}$	$2^{-3} = \dfrac{1}{2^3}$	$\sqrt[n]{a} = a^{1/n}$	$\sqrt[3]{7} = 5^{1/3}$
$a^{1/n} = \sqrt[n]{a}$	$6^{1/5} = \sqrt[5]{6}$	$\dfrac{1}{\sqrt[n]{a}} = a^{-1/n}$	$\dfrac{1}{\sqrt[5]{2}} = 2^{-1/5}$
$a^{-1/n} = \dfrac{1}{\sqrt[n]{a}}$	$5^{-1/4} = \dfrac{1}{\sqrt[4]{5}}$	$\sqrt[n]{a^p} = a^{p/n}$	$\sqrt[3]{5^2} = 5^{2/3}$
$a^{p/n} = \sqrt[n]{a^p}$	$7^{2/3} = \sqrt[3]{7^2}$	$\dfrac{1}{\sqrt[n]{a^p}} = a^{-p/n}$	$\dfrac{1}{\sqrt[4]{7^3}} = 7^{-3/4}$
$a^{-p/n} = \dfrac{1}{\sqrt[n]{a^p}}$	$6^{-3/4} = \dfrac{1}{\sqrt[4]{6^3}}$		

2.10. FRACTIONAL EXPONENT AND ROOTS.

Don't forget the general rule:

$x^{\frac{1}{2}}$ = the square root of x = $\sqrt{x}$

$x^{\frac{1}{4}}$ = The 4th Root of x = $\sqrt[4]{x}$

So we can come up with a general rule:

A fractional exponent like 1/n means the n-th root:	$x^{\frac{1}{n}} = \sqrt[n]{x}$

What About More Complicated Fractions?

What about a fractional exponent like $4^{3/2}$? That is really saying to do a cube (3) and a square root (1/2), in any order. Here is an explanation:

A fraction (like m/n) can be broken into two parts:

- a whole number part (m) , and
- a fraction (1/n) part

So, because m/n = m × (1/n) we can do this:

$$x^{\frac{m}{n}} = x^{\left(m \times \frac{1}{n}\right)} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

And we get this:

A fractional exponent like m/n means: Do the m-th power, then take the n-th root OR Take the n-th root and then do the m-th power	$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $= \left(\sqrt[n]{x}\right)^m$

Also:

$$a^{p/n} = \sqrt[n]{a^p},\ a > 0 \qquad a^{-p/n} = \frac{1}{\sqrt[n]{a^p}},\ a > 0$$

Some examples:

Example 1: What is $4^{3/2}$?

$$\mathbf{4^{3/2}} = 4^{3\times(1/2)} = \sqrt{(4^3)} = \sqrt{(4\times4\times4)} = \sqrt{(64)} = \mathbf{8}$$

or

$$\mathbf{43/2} = 4^{(1/2)\times3} = (\sqrt{4})^3 = (2)^3 = \mathbf{8}$$

Either way we get the same result.

Example 2: What is 274/3 ?

$$\mathbf{27^{4/3}} = 27^{4\times(1/3)} = \sqrt[3]{(27^4)} = \sqrt[3]{(531441)} = \mathbf{81}$$

or

$$\mathbf{27^{4/3}} = 27^{(1/3)\times 4} = (\sqrt[3]{27})^4 = (3)^4 = \mathbf{81}$$

It was certainly easier the 2nd way!

Exercise 1. Write as a root:

a) $\sqrt[5]{3}$ b) $\dfrac{1}{\sqrt[6]{5}}$ c) $\sqrt[7]{3^5}$ d) $\dfrac{1}{\sqrt[3]{7^2}}$

Exercise 2. Write as a root and calculate the result:

a) $27^{1/3} =$ b) $49^{-1/2} =$ c) $128^{3/7}$ d) $243^{-2/5}$

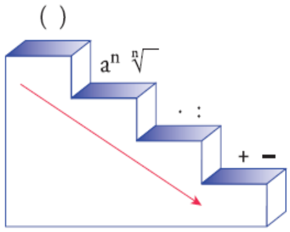
Exercise 3. Use the properties of roots to write these operations with just one root:

a) $\sqrt{5}\cdot\sqrt{3}$ b) $\sqrt{6} : \sqrt{3}$ c) $(\sqrt[3]{5})^2$ d) $\sqrt[3]{\sqrt{5}}$

Exercise 4. Use the properties of roots to calculate the result:

a) $\sqrt{6}\cdot\sqrt{6}$ b) $\sqrt{20} : \sqrt{5}$ c) $\sqrt[3]{25}\cdot\sqrt[3]{5}$ d) $\sqrt{\sqrt[3]{64}}$

REMINDER:

	**1**.Brackets. **2**.Powers and roots. **3**. Multiplications and divisions. **4**. Additions and subtractions.

Example:

$(56 - 7^2)\cdot\sqrt{25} = (56 - 49)\cdot 5 = 7\cdot 5 = 35$

With the calculator: (56 − 7 x²) × √ 25 = **35**

Exercise 5: Work out in your mind and check it with the calculator:

a) $(9^2 + 23 - 7^2)\cdot\sqrt{64}$ ....

b) $(10^2 - \sqrt{81} + 5^3) : \sqrt{36}$ ....

EXERCISES

UNIT 2. POWERS AND ROOTS.

Exercise 1. Fill in the gaps in the table and calculate:

POWER	BASE	EXPONENT
2^5		
A^3		

$2^5 =$ ________
$3^3 =$ ________
$5^3 =$ ________

Exercise 2. Fill in the gaps using the properties of powers:

$2^5 \cdot 5^5 = \boxed{}^{\boxed{}} = \boxed{}$

$28^3 : 7^3 = \boxed{}^{\boxed{}} = \boxed{}$

$(5^7 \cdot 5^3) : 5^6 = 5^{\boxed{}} : 5^{\boxed{}} = 5^{\boxed{}} = \boxed{}$

$3^2 \cdot 3^5 = \boxed{}^{\boxed{}}$

$6^8 : 6^3 = \boxed{}^{\boxed{}}$

$(5^3)^3 = \boxed{}^{\boxed{}}$

$a^4 \cdot a^2 = \boxed{}^{\boxed{}}$

$a^9 : a^6 = \boxed{}^{\boxed{}}$

$(a^3)^2 = \boxed{}^{\boxed{}}$

Exercise 3. Fill in the gaps using the properties of powers:

$(a^3 \cdot a^4) \cdot a^2 = a^{\boxed{}} \cdot a^2 = a^{\boxed{}}$

$(m^7 : m^2) : m^3 = m^{\boxed{}} : m^3 = m^{\boxed{}}$

$(a^3)^2 : a^4 = a^{\boxed{}} : a^4 = a^{\boxed{}}$

$(m^3)^3 : (m^3)^2 = m^{\boxed{}} : m^{\boxed{}} = m^{\boxed{}}$

$a^3 \cdot a^6 = a^{\boxed{}}$ $(a^3)^3 = a^{\boxed{}}$

$m^7 : m^5 = m^{\boxed{}}$ $(m^2)^4 = m^{\boxed{}}$

$b^{10} : b^7 = b^{\boxed{}}$ $(b^3)^2 = b^{\boxed{}}$

Exercise 4. Calculate:

a) $(-2)^0 =$ b) $(-2)^1 =$ c) $(-2)^2 =$ d) $(-2)^3 =$ e) $(-2)^4 =$ f) $(-2)^5 =$

g) $-2^2 =$ h) $-2^3 =$ i) $-3^3 =$

Exercise 5. Rewrite these figures using scientific notation:

a) 25,000,000 b) 56,789,234 c) 0.000000234 d) 0.000893

____________ ____________ ____________ ____________

Exercise 6. Apply the properties of powers:

a) $3^2 \cdot 3^5 =$ _____ b) $3^5 : 3^2 =$ _____ c) $(3^5)^2 =$ _____ d) $2^4 \cdot 5^4 =$ _____ e) $2^7 : 5^7 =$ _____

Exercise 7. Write with integer base:

a) $\frac{1}{2^7}$ b) $\frac{1}{3}$ c) $\frac{1}{7^4}$ d) $\frac{1}{64}$

....

Exercise 8. Write the first five perfect squares bigger than 30:

....

Exercise 9. Approximate the square root of the following numbers:

.... $< \sqrt{23} <$ .... $< \sqrt{62} <$ .... $< \sqrt{56} <$ ....

.... $< \sqrt{44} <$ .... $< \sqrt{93} <$ .... $< \sqrt{120} <$ ....

Exercise 10. Apply the properties of square roots to calculate:

a) $\sqrt{3} \cdot \sqrt{12}$ .... b) $\sqrt{72} : \sqrt{8}$ ....

Exercise 11. Extract the factors of these roots:

a) $\sqrt{20}$ .... b) $\sqrt{75}$ .... c) $\sqrt{98}$ ....

Exercise 12. Calculate using the priority of operations:

a) $\left(7\sqrt{36} - 8^2 + 15\right) \cdot \sqrt{100}$ ....

b) $\left(7^2 + 476 - \sqrt{64} + 2^5\right) : \sqrt{81}$ ....

Exercise 13. Calculate these cube roots:

$a)\sqrt[3]{8} =$ .... $b)\sqrt[3]{-27} =$ .... $c)\sqrt[3]{-125} =$ .... $d)\sqrt[3]{-1000000} =$ ....

Exercise 14. Approximate these cube roots:

.... $< \sqrt[3]{7} <$ .... $< \sqrt[3]{25} <$ .... $< \sqrt[3]{100} <$ ....

.... $< \sqrt[3]{-29} <$ .... $< \sqrt[3]{-59} <$ .... $< \sqrt[3]{-155} <$ ....

Exercise 15. Extract the factors of these cube roots:

a) $\sqrt[3]{56}$ .... b) $\sqrt[3]{135}$ .... c) $\sqrt[3]{6\,000}$ ....

Exercise 16. Calculate applying the properties of powers:

a) $x^5 \cdot x^3 =$ ______ b) $x^4 : x^9 =$ ______ c) $(x^3)^5 =$ ______ d) $x^5 \cdot x^2 : x^3 =$ ______

Exercise 17. What is the value of "x" in each case?

a) $2^x = 32$ ________________________ b) $x^4 = 81$ ________________________

c) $5^3 = x$ ________________________ d) $(-2)^x = 64$ ________________________

Exercise 18. What is the value of "x" in each case?

a) $\sqrt{x} = 7$ .... b) $\sqrt{36} = x$ ....

c) $\sqrt[3]{x} = 4$ .... d) $\sqrt[3]{x} = -5$ ....

Exercise 19. Calculate:

a) $(3 + 4)^2 =$ ______ b) $3^2 + 4^2 =$ ______ c) $(13 - 5)^2 =$ ______ d) $13^2 - 5^2 =$ ______

Exercise 20. Calculate:

a) $\sqrt{9 + 16}$ .... b) $\sqrt{9} + \sqrt{16}$ .... c) $\sqrt{100 - 64}$ .... d) $\sqrt{100} - \sqrt{64}$ ....

Exercise 21. Calculate using the calculator:

a) $\left(13\sqrt{81} - 12^2 + 105\right) \cdot \sqrt{625}$ ....

b) $\left(7^3 - 5\,334 - \sqrt{169} + 2^7\right) : \sqrt[3]{12\,167}$ ....

Exercise 22. Calculate with the calculator and write the result using scientific notation:

a) $2^{64} =$ ________________ b) $5.3 \cdot 10^{23} \cdot 4.81 \cdot 10^{-5} =$ ________________

c) $3^{15} =$ ________________ d) $8.75 \cdot 10^{12} : (6.32 \cdot 10^{-4}) =$ ________________

Exercise 23. Calculate applying the properties of powers:

a) $\dfrac{(a^3)^2}{a^4} =$ ....

b) $\dfrac{(2 \cdot 3)^3}{2^2 \cdot 3^2} =$ ....

c) $a^6 : a^4 =$ ....

d) $(a^3 \cdot a^4) : a^5 =$ ....

e) $(4 \cdot 5)^3 : (4^2 \cdot 5^2) =$ ....

Exercise 24. You need to contact all the members in your football league. You phoned 3 people in the morning. Those 3 people each phoned 3 more people in the afternoon. That night, those additional people each phoned 3 others. How many people got a phone call by then?

Exercise 25. In a cinema, there is the same number of rows and columns, and there are 289 seats in total. How many seats are there in each row?

Exercise 26. The schoolyard is square, and it is 60 metres long. We want to lay paving stones whose size is 40 cm x 40 cm. They cost € 0.65 each, and building workers charge us € 3,000. How much does it cost to repair the schoolyard?

Translate the instructions: ....

....

....

Exercise 27. In a restaurant, they have 5 starters, 5 main dishes and 5 different desserts. How many days can I go to the restaurant without repeating the same menu?
Translate the instructions: ....

....

....

Exercise 27. The charge to cover the four walls of a kitchen with tiles is € 900. If the walls are square and they charge €25/m², how long is each wall?
Translate the instructions: ....

....

Exercise 28. The hard disk of a laptop has 40 Gb. If 1 Gb = 2^{10} Mb, 1 Mb = 2^{10} Kb and 1 Kb = 2^{10} bytes, how many bytes is the capacity of this hard disk? Write it in scientific notation.

Exercise 29. Simplify the roots: a) $\sqrt[6]{7^2}$ b) $\sqrt[15]{7^{12}}$ c) $\sqrt[20]{7^{12}}$ d) $\sqrt[30]{7^{18}}$

Exercise 30. Multiply to eliminate the brackets.

a) $2a^3b\,(3a^2b - 6a^3b^3) =$

b) $3xy^2z^3\,(4x^2y^3z + 5x^3y - 7x^5z) =$

Exercise 31. Extract common factor:

a) $12a^4b^5 - 18a^3b^6 =$

b) $6x^5y^2z^3 + 15x^2y^5z^3 - 18x^2y^3z^5 =$

Exercise 32. Calculate: a) $7^{-1} =$ b) $(-7)^{-1} =$ c) $\left(\frac{5}{3}\right)^{-1}$ d) $\left(\frac{1}{2}\right)^{-1}$

Exercise 33. Calculate in your mind:
a) $3^{-1} =$ b) $(-3)^{-1} =$ c) $3^{-2} =$ d) $(-3)^{-2} =$ e) $3^{-3} =$ f) $(-3)^{-3} =$

Exercise 34. Using the properties of powers, write these as just one power:

a) $3^5 \cdot 3^{-4} =$ b) $2^4 : 2^{-3} =$ c) $(3^{-4})^{-3} =$ d) $17^{-2} \cdot 17^3 \cdot 17^{-4} =$

Exercise 35. Using the properties of roots, calculate:
a) $\sqrt{27} \cdot \sqrt{3}$ b) $\sqrt{45} : \sqrt{5}$ c) $\sqrt[3]{4} \cdot \sqrt[3]{16}$ d) $\sqrt[5]{\sqrt{1\,024}}$

Exercise 36. Calculate the value of "x" in each case:
a) $2^x = 32$
b) $3^4 = x$
c) $x^3 = 125$
d) $x^3 = -8$

Exercise 37. Calculate: a) $\left(\frac{2}{3}\right)^3$ b) $\left(-\frac{2}{3}\right)^3$

c) $\left(\frac{2}{3}\right)^4$ d) $\left(-\frac{2}{3}\right)^4$ e) 5^{-1} f) $(-5)^{-1} =$

g) 2^{2^3} h) $\left(-\frac{1}{3}\right)^{-1}$

Exercise 38. Calculate the value of "x" in each case:
a) $\sqrt{x} = \pm 5$ b) $\sqrt{49} = x$ c) $\sqrt[3]{x} = 5$ d) $\sqrt[x]{32} = 2$

Exercise 39. Calculate using prime factor decomposition:

a) $\sqrt[3]{216}$ b) $\sqrt[3]{3\,375}$ c) $\sqrt[3]{\frac{8}{125}}$ d) $\sqrt[5]{\frac{243}{32}}$

Exercise 40. Extract as many factors as possible:
a) $\sqrt{243a^8b^3c^7}$
b) $\sqrt[3]{125a^9b^{17}c^{25}}$

c) $\sqrt{81a^5bc^6}$

d) $\sqrt[3]{32a^8b^2c^{12}}$

Exercise 41. Calculate the following operations with roots:

a) $3\sqrt{32} - 2\sqrt{50} + \sqrt{72}$

b) $2\sqrt{75} - 4\sqrt{27} + 5\sqrt{12}$

a) $2\sqrt{200} - 3\sqrt{18} - 4\sqrt{98}$

Exercise 42. Fill the gaps using the signs $=$ or $\neq$:

a) 5^3 ... 15 b) $(-6)^5$ ... -6^5 c) $(3^5)^2$ ... 3^{10} d) $(7-5)^4$ ... 16

MATHS IS FUN!