UNIT 1.NUMBERS

1. LET'S REMEMBER SOMETHING ABOUT MULTIPLES AND FACTORS

1.1.Concept of multiple

We say that a number "a" **is a multiple** of another number "b" if the division a : b is an **exact division**, that is, if "b" contains "a" a whole number of times.

(Para obtener los múltiplos de un número lo multiplicamos por 1, 2, 3 y así sucesivamente.)

1.2. Concept of factor

We say that a number "b" **is a factor** of another number "a" if the division a : b is an exact division.

Solve the following exercises:

1. Find three multiples of 11 that are between 32 and 99.

2. Work out if 556 is a multiple of 6.

3. Find out if 12 is a factor of 1520.

4. Point out which of these numbers have exactly three factors.

a) 4

b) 25

c) 15

d) 49

2. PRIME AND COMPOSITE NUMBERS

So, a **prime number** only has two factors: the number one and itself. For example: 3, 5, 11, 17, etc.

A **composite number** has more than two factors. For example: 4, 9, 15, 30, etc.

A smart procedure to find the first prime numbers is the Sieve of Erathostenes. It consists of a table with the numbers from 1 to 100, like the one below, and now follow the following rules:

- Number 2 is prime. Circle it, then cross out all the multiples of 2
- Circle the next number that is not crossed out (3) because it is prime too. And then, cross out all its multiples.
- Continue in this way, that is, circle the numbers which are not crossed out and cross out all their multiples until you finish with the table. Then you will have the first prime numbers under 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solve the following exercise:

Work out the factors of the numbers below and then point out which ones are prime numbers:

a) 8 b) 101 c) 57 d) 49

3.- DIVISIBILITY RULES

These rules help you to know if a number is a multiple of another without doing the division:

- Rule of number 2: A number is divisible by 2 if its last digit is either 0 or an even number. Un número es divisible por 2 si su última cifra es 0 ó un número par. Example: 46, 200, 34, 108.....
- Rule of number 3: A number is divisible by 3 if the sum of its digits is a multiple of 3. Un número es divisible por 3 si la suma de sus cifras es múltiplo de 3. Example: 45, 105, 300, 417....
- Rule of number 4: A number is divisible by 4 if its two last digits are multiples of 4. Un número es divisible por 4 si sus dos últimas cifras son múltiplo de 4. Example: 100, 224, 340, 664....
- Rule of number 5: A number is divisible by 5 if it ends in 0 or 5. Un número es múltiplo de 5 si acaba en 0 ó 5. Example: 200, 345, 650, 800 .....
- Rule of number 9: A number is divisible by 9 if the sum of its digits is a multiple of 9. Un número es divisible por 9 si la suma de sus cifras es múltiplo de 9. Example: 81, 333, 450, 1278.....
- Rule of number 10: A number is divisible by 10 if it ends in 0. Un número es divisible por 10 si acaba en 0. Example: 30, 400, 500.
- Rule of number 11: A number is divisible by 11 if the difference between the sum of the digits on odd positions and the sum of the digits on even positions is 0, 11 or a multiple of 11. Un número es divisible por 11 si la diferencia entre la suma de las cifras en posición par y la suma de las cifras en posición impar es 0, 11 o un múltiplo de 11. Example: 121, 3652

Solve the following exercises:

1. Use the divisibility rules to complete the following table:

Divisible by	2	3	4	5	9	10	11	25	100
375									
990									
1.848									
12.300									
14.240									

2. Find out two numbers with five digits that are divisible by both 2 and 5 and aren't divisible by 100.

4. THE HIGHEST COMMON FACTOR AND THE LEAST COMMON MULTIPLE

4.1. Concept of the highest common factor (HCF)

To understand the concept, check this example to calculate HCF (30, 48, 54).

Firstly, calculate:

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Now, we choose the common factors: (Ahora elegimos los divisores comunes a los tres números)

Factors of 30: **1**, **2**, **3**, 5, **6**, 10, 15, 30

Factors of 48: **1**, **2**, **3**, 4, **6**, 8, 12, 16, 24, 48

Factors of 54: **1**, **2**, **3**, **6**, 9, 18, 27, 54

Which is the biggest one? It is 6, so:

HCF (30, 48, 54) = 6

Definition:

The highest (or greatest) common factor of several numbers is the largest number that evenly divides into

all of them.

4.2. Rule for calculating the H.C.F.

"To work out the HCF of several numbers, first you have to find the prime factor decomposition of the given numbers and then, take the common factors with the lowest index".

Solved example:

Find out the HCF of numbers 36, 48 and 90.

1^{st} step: Write them as a product of prime factors:

$36 = 2^2 \cdot 3^2$ $\quad 48 = 2^4 \cdot 3$ $\quad 90 = 2 \cdot 3^2 \cdot 5$

2^{nd} step: Take the common factors with the lowest index:

H.C.F. = 2 · 3 = 6

We can also do it in the English way. It consists in writing all the factors of each number in a row, and then mark the common ones.

36 = 2 · 2 · 3 · 3

48 = 2 · 2 · 2 · 2 · 3

90 = 2 · 3 · 3 ·5

We mark the factors that are common in all three numbers:

Spanish: Señalamos los factores que sean comunes en los tres números:

36 = **2** · 2 · **3** · 3

48 = **2** · 2 · 2 · 2 · **3** ⟹ H.C.F. = 2 · 3 = 6

90 = **2** · **3** · 3 · 5

Solve the following exercises:

1. Work out IN YOUR MIND the factors of the numbers below and then find out the HCF:

a) 2 and 16

b) 3 and 25

c) 9, 12 and 18

d) 27, 36 and 63

2. Find out the HCF of the following numbers IN YOUR MIND:

a) 4, 6, 18 and 32

b) 3, 4, 12, 36 and 48

4.3. Concept of the least common multiple (lcm)

In this case, we calculate l.c.m. (2,3):

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,...

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36,....

Now, we choose the common multiples:

Multiples of 2: 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, 20, 22, **24**, 26, 28, **30**, 32,...

Multiples of 3: 3, **6**, 9, **12**, 15, **18**, 21, **24**, 27, **30**, 33, 36,....

Which is the smallest? It is 6, so:

l.c.m. (2,3)= 6

Definition: The least (or lowest) common multiple of several numbers is the smallest number that is multiple of all of them.

4.4. Rule for calculating the l.c.m.

"To work out the lcm of several numbers, first write them as a product of their prime factors and then take the common and non-common factors with the highest index."

Solved example:

Find out the lcm of numbers 36, 48 and 90:

1^{st} step: First obtain the prime factor decomposition:

$36 = 2^2 \cdot 3^2$ $48 = 2^4 \cdot 3$ $90 = 2 \cdot 3^2 \cdot 5$

2^{nd} step: Now, take the common and non-common factors with the highest index:

$$l.c.m = 2^4 \cdot 3^2 \cdot 5 = 720$$

Solved example:

Find out mentally the HCF and the lcm of the numbers below:

a) 3 and 5; HCF = 1 and lcm = 15

b) 2 and 4; HCF = 2 and lcm = 4

c) 6 and 15; HCF = 3 and lcm = 30

Solve the following exercises:

1. Work out IN YOUR MIND the l.c.m. of the numbers below:

a) 9, 12 and 18 b) 27, 36 and 63

2. Work out the l.c.m. of the following numbers. What conclusion do you reach?

a) 2, 4, 8 and 16 b) 3, 4, 6 and 12

5. REVISING INTEGERS.

We will remember all about fractions in the following section but first, we´ll revise some operations with integers numbers.

Do you remember how to calculate the following operations?

a) $32 + (-12) : 6 =$

b) $(-8) \times 9 - 15 \times (-3) =$

c) $(-4) \times 10 : 2 + 14 : (-7) =$

d) $27 : (-3) \times 2 - (-4) =$

e) $(-18) : 6 + 5 \times (-10) =$

f) $18 : 9 + 5 - [(-15) \times 3 + 12 \times 4] =$

g) $(-6) \times [4 - (-2)] + [-8 + (-3) \times 2] =$

h) $(-35) : (5 + 2) + (-4) \times 9 - (7 - 2 \times 5) =$

i) $[(3 - 4) + (-2)] \times 4 + 9 : (-3) \times 6 =$

j) $\{[(-12) - (-3)] \times 8\} + 24 : \{[(-2) + (-6)] : 2\} =$

The following lists some of the basic properties of addition and multiplication for any integers:

Properties	Addition		Multiplication
Associative:	$(a+b)+c = a+(b+c)$		$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Commutative:	$a+b = b+a$		$a \cdot b = b \cdot a$
Existence of neutral element:	$0 \in \mathbb{Z}$ / $a+0 = a$		$1 \in \mathbb{Z}$ / $a \cdot 1 = a$
Existence of inverse element:	$a+(-a)=0$; $-a$ is the opposite of a		
Distributive:	$a \cdot (b+c) = a \cdot b+a \cdot c$		

The existence of inverse elements for addition allows us to define the subtraction of two integer numbers by adding the opposite of the second number to the first one:

$$a - b = a + (-b)$$

So, the addition and the subtraction are closed operations in $\mathbb{Z}$: $a \pm b$ are always integer.

Examples: $(+3)-(+5)=+3-5=-2$ $(-4)-(-5)=-4+5=+1$

But not every integer has a multiplicative inverse; e. g. there is no integer e such that $2 \cdot e=1$, because the left hand side is even, while the right hand side is odd. The number e have to be $\frac{1}{2}$, which is not integer, but rational.

In other words, the division of two integer numbers is not always an integer.

The multiplication of two integer numbers is a closed operation in $\mathbb{Z}$: $a \cdot b$ is always integer (but not the division).

To divide integers you have to apply the same rules as the multiplication.

Examples:	$(-5)\cdot(+3)=-15$	$(-20):(+4)=-5$
	$(-4)\cdot(-6)=+24$	$(-32):(-8)=+4$

Let's revise how to do operations with fractions.

5.1. ADD AND SUBTRACT FRACTIONS.

When the denominators are the same. Easy! Try it alone! Write one example!

When the denominators are different.

Let's try this: $\frac{1}{2}+\frac{1}{3}$

The main rule of this game is that we can't do anything until the denominators are the same!

We need to find something called the least common denominator (LCD)... It's really just the LCM of our denominators, 2 and 3.

The LCM of 2 and 3 is 6. So, our LCD is 6.

We need to make this our new denominator...

Change the $\frac{1}{2}$: $\frac{1\times 3}{2\times 3}=\frac{3}{6}$

Change the $\frac{1}{3}$: $\frac{1\times 2}{3\times 2}=\frac{2}{6}$

So $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$ Now we can do it! $\frac{3+2}{6}=\frac{5}{6}$

Summing up:

How to add or subtract fractions with different denominators:

Find the Least Common Denominator (LCD) of the fractions
Rename the fractions to have the LCD
Add or subtract the numerators of the fractions
Simplify the Fraction

5.2. How to Multiply Fractions.

This is easy!

We just multiply straight across...

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

Then just reduce it

5.3. How to Divide Fractions.

This is easy too, we just do cross-multiplication...

$$\frac{1}{3} \div \frac{4}{5} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$$

5.4. Powers of Fractions.

Simply multiply the numerator and the denominator by themselves as many times as the power (exponent) indicates.

$$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

5.5. Order of operations.

The order of operations for fractions is the same as that for natural numbers or integers.

Remember: BODMAS.
B = Brackets.
O = Orders (Exponents or indexes)
DM = Division and Multiplication.
AS = Addition and Subtraction.

If you want to learn this rule with a song, you can visit this video:

http://www.youtube.com/watch?v=TMTESTKAjPg

EXERCISES

1. Calculate IN YOUR MIND:
   a) 2/3 of 12 =
   b) 4/7 of 35 =
   c) 3/8 of 80 =
   d) 7/5 of 135 =

2. Compare the following fractions: 2/3, 23/4, 5/5. Which is the biggest? Which is the smallest?

3. Draw the following numbers on the real line: $\frac{1}{2}$, $\frac{3}{4}$, 7/3, 11/4, 7/2, 14/3.

4. Calculate in your mind:

   a) $\frac{1}{2} + 1$ b) $2 - \frac{1}{3}$ c) $2 \cdot \frac{3}{5}$

5. Calculate the inverse fraction and the opposite fraction of:

   $\frac{2}{3}, -\frac{4}{5}, \frac{1}{2}, -\frac{1}{3}$

6. Work out:

a) $\frac{1}{3} - \frac{5}{6} + \frac{3}{2}$

b) $\frac{7}{9} + \frac{11}{12} - \frac{5}{6}$

c) $\frac{3}{8} - 2 + \frac{5}{6}$

d) $\frac{2}{35} + \frac{8}{7} - \frac{3}{10}$

7. Work out:

a) $\frac{2}{9} \cdot \frac{15}{4}$ b) $5 \cdot \frac{3}{25}$ c) $\frac{7}{12} : \frac{3}{4}$

8. Work out:

a) $3 - \left(\frac{1}{4} + \frac{5}{3}\right)$

b) $1 - \left(\frac{3}{2} - \frac{7}{5}\right)$

c) $\frac{2}{3} \cdot \left(\frac{1}{3} + \frac{4}{9}\right)$

d) $\left(\frac{1}{2} - \frac{3}{8}\right) : \frac{3}{4}$

9. Calculate.

a) $\left(\frac{2}{5} - 1\right) \cdot \frac{5}{3}$

b) $\left(\frac{4}{5} - 2\right) \cdot \left(\frac{7}{4} - \frac{5}{6}\right)$

c) $\left(\frac{4}{3} - 2\right) \cdot \frac{3}{10}$

d) $\left(2 - \frac{7}{6}\right) \cdot \left(4 - \frac{7}{3}\right)$

Solutions: a) -1 b) -11/10 c) -1 / 5 d) 25 /18

10. Work out:

a) $\frac{1}{4} + 5 - \left(\frac{5}{8} + \frac{3}{2}\right)$

b) $3 - \left(\frac{5}{6} - 2\right) + \frac{2}{3}$

c) $\frac{1}{2} : \left(\frac{2}{5} - \frac{3}{10}\right)$

d) $3 - \left(\frac{5}{2} - \frac{7}{4}\right) : \frac{3}{2}$

Solutions: a) 25/8 b) 29/6 c) 5 d) 5/2

11. A lorry can transport 12 000 kg and it carries 3/5 of the load. How many kilograms does it carry?

12. We extract 1/6 of a deposit with 1 500 l. Then, we extract 750 l more. What fraction does it left?

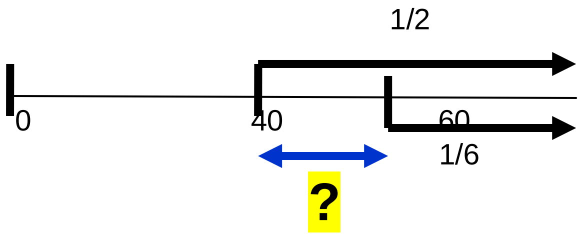
13. Vicente moves forward by ·3/4 of a metre with each step. How many steps does he need to take to cover 30 metres?

14. How many 1/10 litre bottles of perfume can you fill with three half-litre bottles?

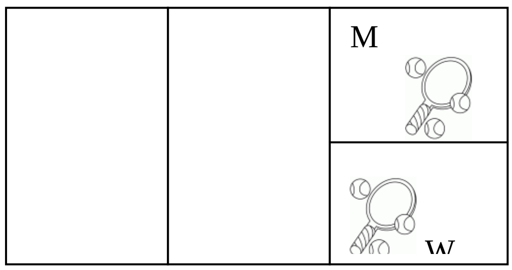
15. 450 students go to a school 45 of them are in the chess club. What fraction of the school´s total students are in the chess club?

16. A travelling salesman bought 800 kilos of melons and has sold three quarters of them. How many kilos does he have left?

17. In a small town, half of the population is over 40 years old and one sixth is over 60. What fraction of the total population is between 40 and 60 years old?



18. One third of a sports club's members play tennis. Half of all the tennis players are women. What fraction of the club's members are women who play tennis?



19. Half a litre of milk is poured evenly into three identical glasses. What fraction of a litre does each glass hold?

6. FRACTION TO DECIMAL CONVERSION.

To convert a fraction into a decimal number, you have to do the division. That´s all!

Example 1:

período

$$\frac{71}{11} = 6,4545\ldots = 6,\widehat{45}$$

Example 2:

período

$$\frac{31}{12} = 2,58333\ldots = 2,58\widehat{3}$$

anteperíodo

Do you remember the types of decimal numbers?
Examples:

- 1.45 → ______________________________
- 5. 33333….. → _______________________________
- -3.56767676767…… → ___________________________
- 0.1010010001000010000001…. ----→

Write more examples:

Look at this table

Important Note: any span of numbers that is underlined signifies that those numbers are repeated. For example, 0.09 signifies 0.090909....
Only irreducible fractions are listed. For instance, to find 2/8, first simplify it to 1/4 then search for it in the table below.

fraction = decimal			
1/1 = 1			
1/2 = 0.5			
1/3 = $0.\underline{3}$	2/3 = $0.\underline{6}$		
1/4 = 0.25	3/4 = 0.75		
1/5 = 0.2	2/5 = 0.4	3/5 = 0.6	4/5 = 0.8
1/6 = $0.1\underline{6}$	5/6 = $0.8\underline{3}$		
1/7 = $0.\underline{142857}$	2/7 = $0.\underline{285714}$	3/7 = $0.\underline{428571}$	4/7 = $0.\underline{571428}$
	5/7 = $0.\underline{714285}$	6/7 = $0.\underline{857142}$	
1/8 = 0.125	3/8 = 0.375	5/8 = 0.625	7/8 = 0.875
1/9 = $0.\underline{1}$	2/9 = $0.\underline{2}$	4/9 = $0.\underline{4}$	5/9 = $0.\underline{5}$
	7/9 = $0.\underline{7}$	8/9 = $0.\underline{8}$	
1/10 = 0.1	3/10 = 0.3	7/10 = 0.7	9/10 = 0.9

1/11 = $0.\underline{09}$	2/11 = $0.\underline{18}$	3/11 = $0.\underline{27}$	4/11 = $0.\underline{36}$
	5/11 = $0.\underline{45}$	6/11 = $0.\underline{54}$	7/11 = $0.\underline{63}$
	8/11 = $0.\underline{72}$	9/11 = $0.\underline{81}$	10/11 = $0.\underline{90}$
1/12 = $0.08\underline{3}$	5/12 = $0.41\underline{6}$	7/12 = $0.58\underline{3}$	11/12 = $0.91\underline{6}$
1/16 = 0.0625	3/16 = 0.1875	5/16 = 0.3125	7/16 = 0.4375

7. DECIMAL NUMBER TO FRACTION CONVERSION.

- **EXACT DECIMAL.** Look at the example:

	USING THE CALCULATOR
$2{,}75 = \frac{275}{100} = \frac{11}{4}$ Simplifying	$\boxed{2.75}\ \boxed{=}\ \boxed{a^{b}/_{c}}$ **11 ⌟ 4**
2.405 = ———	Try with your calculator

- **REPEATING DECIMALS.** Look at the example

	USING THE CALCULATOR
$5{,}\widehat{3} = \frac{53-5}{9} = \frac{48}{9} = \frac{16}{3}$ Simplifying	$\boxed{(}\ \boxed{53}\ \boxed{-}\ \boxed{5}\ \boxed{)}\ \boxed{a^{b}/_{c}}$ $\boxed{9}\ \boxed{=}$ **16 ⌟ 3**
1.232323...= ————	Try with your calculator

- **REPEATING DECIMALS THAT BEGIN WITH A NON-REPEATING PART.**

	USING THE CALCULATOR
$2{,}3\widehat{18} = \frac{2318-23}{990} = \frac{2295}{990} = \frac{51}{22}$ Simplifying	$\boxed{(}\ \boxed{2318}\ \boxed{-}\ \boxed{23}\ \boxed{)}\ \boxed{a^{b}/_{c}}$ $\boxed{990}\ \boxed{=}$ **51 ⌟ 22**
0.21456456456456456...= ————	Try with your calculator

8. REAL NUMBERS.

This classification of Numbers represents the most accepted elementary classification.

Class	Symbol	Description
Natural Number	$\mathbb{N}$	Natural numbers are defined as non-negative counting numbers: $\mathbb{N}$ = { 0, 1, 2, 3, 4, ... }. Some exclude 0 (zero) from the set: $\mathbb{N}$ * = $\mathbb{N}$ \{0} = { 1, 2, 3, 4, ... }.
Integer	$\mathbb{Z}$	Integers extend $\mathbb{N}$ by including the negative of counting numbers: $\mathbb{Z}$ = { ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ... }. The symbol $\mathbb{Z}$ stands for Zahlen, the German word for "numbers".
Rational Number	$\mathbb{Q}$	A rational number is the ratio or quotient of an integer and another non-zero integer: $\mathbb{Q}$ = {n/m | n, m ∈ $\mathbb{Z}$, m ≠ 0 }. E.g.: -100, -20¼, -1.5, 0, 1, 1.5, 1½ 2¾, 1.75, &c
Irrational Number		Irrational numbers are numbers which cannot be represented as fractions. E.g.: √2, √3;, π, e.
Real Number	$\mathbb{R}$	Real numbers are all numbers on a number line. The set of $\mathbb{R}$ is the union of all rational numbers and all irrational numbers.
Imaginary Number		An imaginary number is a number whose square is a negative real number, and is denoted by the symbol i, so that $I^2 = -1$. E.g.: -5i, 3i, 7.5i, etc. In some technical applications, j is used as the symbol for an imaginary number instead of i.
Complex Number	$\mathbb{C}$	A complex number consists of two parts, a real number and an imaginary number, and is also expressed in the form a + bi (i is the notation for the imaginary part of the number). E.g.: 7 + 2i

❖ Rational Numbers

A **Rational** Number can be written as a **Ratio** of two integers (ie a simple fraction).

Example: **1.5** is rational, because it can be written as the ratio **3/2**

Example: **7** is rational, because it can be written as the ratio **7/1**

Example **0.317** is rational

❖ Irrational Numbers

Some numbers **cannot** be written as a ratio of two integers...

...they are called **Irrational Numbers**.

?	*They are called **irrational** because they cannot be written as a **ratio** (or fraction), not because they are crazy!*

Example: **π** (Pi) is an irrational number.

	**π = 3.14159265358979323846264338327950 (and more...)** You **cannot** write down a simple fraction that equals Pi.

The popular approximation of $^{22}/_7$ = 3.1428571428571... is close but **not accurate**.

Another clue is that the decimal goes on forever without repeating.

Rational vs Irrational

So you can tell the difference between Rational and Irrational by trying to write the number as a simple fraction.

Example: 9.5 can be written as a simple fraction like this:

$$9.5 = {}^{19}/_2$$

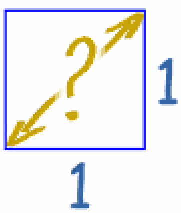
So it is a **rational number** (and so it is **not irrational**)

Here are some more examples:

Number	As a Fraction	Rational or Irrational?
5	5/1	Rational
1.75	7/4	Rational
.001	1/1000	Rational
√2 (square root of 2)	?	**Irrational !**

Square Root of 2

Let's look at the square root of 2 more closely.

	If you draw a square (of size "1"), what is the distance across the diagonal?

The answer is the **square root of 2**, which is **1.4142135623730950...(etc)**

But it is not a number like 3, or five-thirds, or anything like that ...

... in fact you **cannot** write the square root of 2 using a ratio of two numbers.

If you would like to know **why**, have a look on this webpage:

http://www.mathsisfun.com/numbers/irrational-finding.html

... and so we know it is **an irrational number**

Famous Irrational Numbers

π	**Pi** is a famous irrational number. People have calculated Pi to over one million decimal places and still there is no pattern. The first few digits look like this: 3.1415926535897932384626433832795 (and more ...)
e	The number ***e*** (Euler's Number) is another famous irrational number. People have also calculated ***e*** to lots of decimal places, and they have not found any pattern. The first few digits look like this: 2.7182818284590452353602874713527 (and more ...)
φ	The Golden Ratio is an irrational number. The first few digits look like this: 1.61803398874989484820... (and more ...)
$\sqrt{}$	Many square roots, cube roots, etc. are also irrational numbers. Examples: √3 1.7320508075688772935274463415059 (etc) √99 9.9498743710661995473447982100121 (etc)
But √4 = 2 (rational), and √9 = 3 (rational) ... ... so not all roots are irrational.	

History of Irrational Numbers

Apparently Hippasus (one of Pythagoras' students) discovered irrational numbers when he was trying to represent the square root of 2 as a fraction (using geometry, probably). Instead he proved that you cannot write the square root of 2 as a fraction, and so it was irrational.

However, Pythagoras could not accept the existence of irrational numbers, because he believed that all numbers had perfect values. But he could not disprove Hippasus' "irrational numbers" and so Hippasus was thrown overboard and drowned!

QUESTION 1. Which one of the following is not irrational? $\sqrt{2}$, $\sqrt{4}$, $\sqrt{3}$, $\sqrt{5}$.

QUESTION 2. Draw the classification of numbers according to the following diagrams:

Diagram 1:

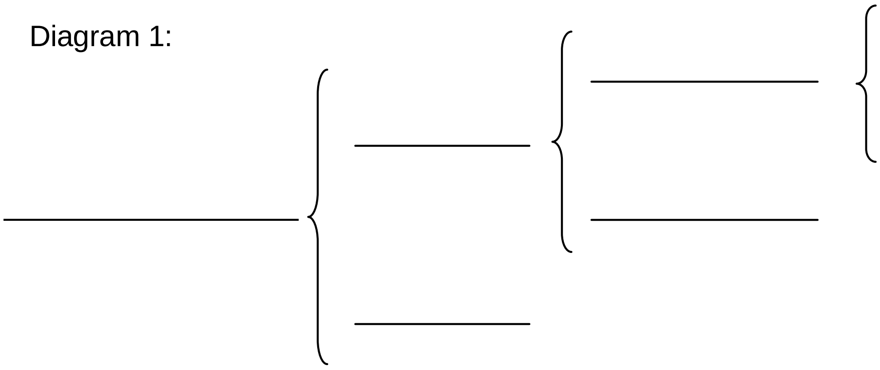
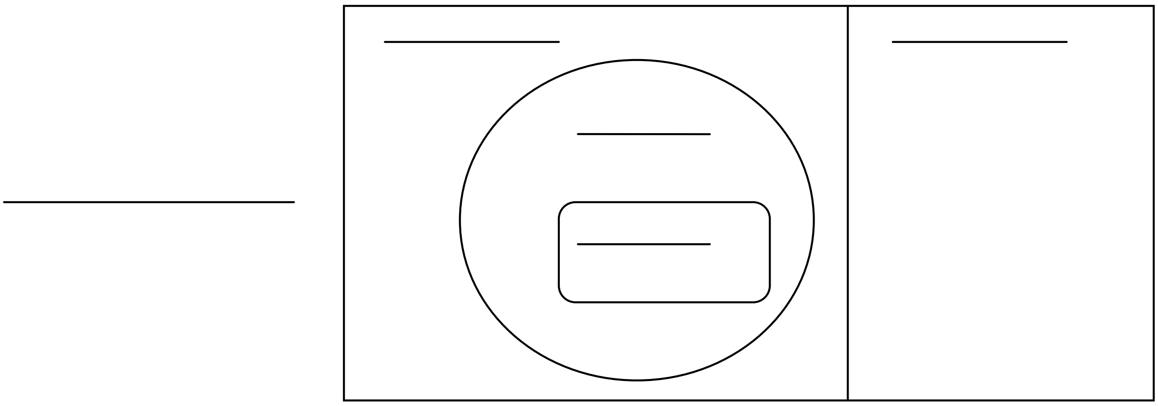


Diagram 2:



9. ROUNDING NUMBERS.

What is "Rounding"? *Rounding means reducing the digits in a number while trying to keep its value similar.* The result is less accurate, but easier to use.

Example: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

Common Method

There are several different methods for rounding, but here we will only look at the **common method**, the one used by most people.

How to Round Numbers

- Decide which is the last digit to **keep**
- Leave it the same if the **next digit** is less than 5 (this is called *rounding down*)
- But increase it by 1 if the next digit is 5 or more (this is called *rounding up*)

Example: Round 74 to the nearest 10

- We want to keep the "7" as it is in the 10s position
- The next digit is "4" which is less than 5, so no change is needed to "7"

So, the answer is 70 .

(74 gets "rounded down")

Example: Round 86 to the nearest 10

- We want to keep the "8"
- The next digit is "6" which is 5 or more, so increase the "8" by 1 to "9"

So the answer is 90.

(86 gets "rounded up")

So: when the first digit **removed** is 5 or more, increase the last digit **remaining** by 1.

More examples: Rounding to the hundredths (two decimal numbers)

a) $4,52385 = 4,52$ b) $3,34813 = 3,35$

c) $0,62543 = 0,63$ d) $18,59732 = 18,60$

WARNING: In this case we can say, rounding to three significant digits.

10. ABSOLUTE ERROR.

When we round a number we are making a mistake, an error.
The **absolute error** is the absolute value of the difference between the exact value and the rounding value.

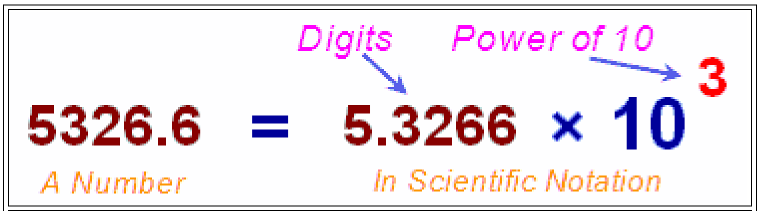
EXAMPLE: If we round $\sqrt{5} = 2,236067\ldots$ to two decimal numbers (to the hundredths) we have got $2,24$, so the absolute error is:

$$\left|\sqrt{5} - 2,24\right| = \left|2,236067\ldots - 2,24\right| = \left|-0,003932\ldots\right| = 0,003932\ldots$$

11. SCIENTIFIC NOTATION / STANDARD FORM

In **Scientific Notation** (also called Standard Form) a number is written in two parts:

- Just the **digits** (with the decimal point placed after the first digit), followed by
- **× 10 to a power** that would put the decimal point back where it should be (i.e. it shows how many places to move the decimal point).



In this example, 5326.6 is written as $\mathbf{5.3266 \times 10^3}$, because $5326.6 = 5.3266 \times 1000 = 5326.6 \times 10^3$

Try it yourself:

http://www.mathsisfun.com/numbers/scientific-notation.html

How to Do it

To figure out the power of 10, think "how many places do I move the decimal point?"

	If the number is 10 or greater, the decimal place has to move **to the left**, and the power of 10 will be positive.
	If the number is smaller than 1, then decimal place has to move **to the right**, so the power of 10 will be negative:
	Example: 0.0055 would be written as $\mathbf{5.5 \times 10^{-3}}$, because $0.0055 = 5.5 \times 0.001 = 5.5 \times 10^{-3}$

Check

After putting the number in Scientific Notation, just check that:

- The "digits" part is between 1 and 10 (it can be 1, but never 10)
- The "power" part shows exactly how many places you moved the decimal point

More examples:

http://www.mathsisfun.com/numbers/scientific-notation.html

Why Use It?

Because it makes it easier when you are dealing with very big or very small numbers, which are common in Scientific and Engineering work.

For example it is easier to write (and read) 1.3×10^{-9} than 0.0000000013.

And it can make calculations easier, as in this example:

Example: a tiny space inside a computer chip has been measured and it is 0.00000256 m wide, 0.00000014 m long and 0.000275 m high.

What is its volume?

Let's first convert the three lengths into scientific notation:

- width: 0.000 002 56m = 2.56×10^{-6}
- length: 0.000 000 14m = 1.4×10^{-7}
- height: 0.000 275m = 2.75×10^{-4}

Then multiply the digits together (ignoring the ×10s):

$2.56 \times 1.4 \times 2.75 = 9.856$

Last, multiply the ×10s:

$10^{-6} \times 10^{-7} \times 10^{-4} = 10^{-17}$ (this was easy: I just added -6, -4 and -7 together)

The result is $\mathbf{9.856\times10^{-17}\ m^3}$

Engineering Notation

Engineering Notation is like Scientific Notation, except that you only use powers of ten that are multiples of 3 (such as 10^3, 10^{-3}, 10^{12} etc).

Example: 19,300 would be written as $\mathbf{19.3 \times 10^3}$

Example: 0.00012 would be written as $\mathbf{120 \times 10^{-6}}$

Notice that the "digits" part can now be between 1 and 1,000 (it can be 1, but never 1,000).

The advantage is that you can replace the **×10**s with Metric Numbers. So you can use standard words (such as thousand or million) prefixes (such as kilo, mega) or the symbol (k, M, etc)

Example: 19,300 meters would be written as **19.3×10^3 m, or 19.3 km**

Example: 0.00012 seconds would be written as **120×10^{-6} s, or 120 microseconds**

EXERCISES

UNIT 1. NUMBERS.

Exercise 1: Indicate which of the following numbers: 35, 39, 84, 375, 420, 570 and 840 are divisible by:

a) 2 and 3 b) 2 and 5.

Exercise 2: Write two numbers which are divisible by 3 and 5.

Exercise 3: Calculate G.C.F. of :

a) 28 and 360 b) 75, 120 and 210

Exercise 4: Calculate l.c.m of:

a) 50 and 140 b) 48, 160 and 300.

Exercise 5: Calculate:

a) $\left(2-\frac{5}{7}\right)\frac{14}{3}$

b) $\left(\frac{3}{5}+3\right)\left(2-\frac{2}{3}\right)$

c) $\left(\frac{5}{6}-\frac{3}{2}\right):\left(\frac{3}{4}-\frac{5}{3}\right)$

d) $\left(2-\frac{5}{6}\right):\left(\frac{3}{4}+\frac{2}{3}\right)$

Exercise 6: Calculate:

a) $\frac{2}{3}:\left(\frac{3}{2}+\frac{5}{6}\right)\left(\frac{7}{6}-2+\frac{1}{3}\right)$

b) $\frac{3}{7}-\left(\frac{1}{3}-\frac{5}{6}\right):\left(\frac{3}{4}+1-\frac{7}{6}\right)$

c) $\left(\frac{5}{6}-\frac{3}{4}\right)\left(\frac{3}{4}+5-\frac{1}{2}\right)$

d) $\frac{4}{9}\left(\frac{2}{3}+2-\frac{1}{6}\right):\frac{2}{9}$

Exercise 7: Convert into fractions the following decimal numbers:

a) $9,\overset{\frown}{692307}$ b) $6,91\overset{\frown}{6}$ c) $1,75$

Exercise 8: Convert into decimal numbers the following decimal fractions:

a) $\dfrac{51}{16}$ b) $\dfrac{36}{11}$ c) $\dfrac{13}{6}$

Exercise 9: Round the two decimal numbers and calculate:

a) $23,567 + 0,413 - 12,085$

b) $0,624 \cdot 1,368$

c) $5,575 : 8,361$

d) $28,508 + 12,534 : 4,197$

Exercise 9: Calculate the absolute error if you round to two decimal numbers:

a) 6,4135 b) 0,0785

c) 4,9084 d) 7,0985

Exercise 10: Using scientific notation, calculate:

$a)\ 3\cdot10^3 + 2\cdot10^2 =$

$b)\ 13\cdot10^4 + 2\cdot10^5 =$

$c)\ 5\cdot10^5 - 25\cdot10^4 =$

$d)\ 3.2\cdot10^6 \cdot 2\cdot10^2 =$

$e)\ 1.2\cdot10^{-6} \cdot 3\cdot10^2 =$

$d)\ 5.2\cdot10^7 : 2\cdot10^{-2} =$

e) $2,62 \cdot 10^{-24} - 7,53 \cdot 10^{-23} =$

f) $5,74 \cdot 10^{11} + 6,5 \cdot 10^{12} =$

e) Using the calculator: $3,85 \cdot 10^{-15} : (3,5 \cdot 10^{-29})$