Unit 7. POLYNOMIALS.

1. WHAT IS ALGEBRA? WHY STUDY ALGEBRA?

To answer this question, please visit this webpage:

http://math.about.com/od/algebra/a/WhyAlgebra.htm

2. ALGEBRAIC LANGUAGE.

Algebraic language (or algebra language) is a language that uses numbers, letters and brackets, connected with operations. It transmits information. It is used in maths and other sciences, and it replaces natural language.

Example:

NATURAL LANGUAGE	ALGEBRAIC LANGUAGE
In a garden, last year, there were **several** trees, and this year, we have planted **five** more trees. How many trees are there this year? Answer: **5 more than last year**.	x 5 x + 5

Exercise 1:

▲△△ Haz corresponder cada enunciado con su expresión algebraica:

- La mitad de un número.
- El triple de la mitad de un número.
- La distancia recorrida en x horas por un tren que va a 60 km/h.
- El precio de x kilos de naranjas que están a 1,3 €/kilo.
- La edad de Pedro, sabiendo que su abuelo, que ahora tiene x años, tenía 60 años cuando nació Pedro.
- El área de un triángulo de base 1,3 m y altura x metros.

$1{,}3x$

$\frac{3x}{2}$

$\frac{x}{2}$

$x - 60$

$\frac{1{,}3x}{2}$

$60x$

Exercise 2:

Complete the table according to the following data:

· Helen is x years old.

· Her daughter is 25 years younger than her.

· Helen's mother's age is double Helen's age.

· Helen's father is 6 years older than her mother.

· Helen was 8 years old when her brother Lawrence was born.

	AGE
Helen	X
Helen's daughter	
Helen's mother	
Helen's father	
Helen's brother	

Exercise 3: Read the information and complete the table:

• Mary gets X euros from her parents every week.

• Susan gets ten less euros than double what Mary gets.
(= A Susan le faltan 10 euros para recibir el doble que Mary.)

• Karen receives fifty euros more than Mary.

	Weekly pay
Mary	X
Susan	
Karen	
All three	

Exercise 4.
There were *f* crocodiles in a river. 68 of the crocodiles get out of the water. Choose the expression that shows how many crocodiles there are now in the river.

a) f − 68 b) f + 68 c) f : 68 d) 68 e) 68 + f

3. POLYNOMIALS.

Polynomials are algebraic expressions that include real numbers and variables. For example: $\mathbf{x^2 - 7x - 6}$

Each part (separated by a + or – sign) is a term, and x^2 is called the leading term because it has the highest exponent.

Monomials contain only one term. For example, **5x,** or **5y**, or $\mathbf{-8x^2}$, or **3** are monomials. A polynomial is the sum of several monomials, so it contains more than one term.

Binomials have two terms, for example: $\mathbf{-3x^2 + 2}$, or $\mathbf{-2y^2 + 9y}$
Trinomials have 3 terms, for example: $\mathbf{-3x^2 + 3x + 2}$, or $\mathbf{-2y^2 + 9y + 5}$

The **coefficient** of the term is the number that is multiplied by the variable, so the coefficient of $4x^2$ is 4.

The **literal part** of a term is the variable or variables, so the literal part of $3x^2y$ is x^2y.
Like terms are terms which have the same literal part, for example $3x^2$ and $-5x^2$.

The **degree of a term** is the number of times a variable is multiplied, so if there is only one variable, the degree is its exponent: $3x^2$ has a degree of 2.

The **degree of a polynomial** is the highest degree of its terms.

When there is more than one variable, we add up the exponents, and the degree increases accordingly, so the degree of 3xy is 2 (1+1), and the degree of $4x^2y^3$ is 5 (2+3).
When the variable does not have an exponent, we understand that there is a '1': $3x = 3x^1$, so the degree of 3x is 1.

Term	Numerical Coefficient	Literal part	Degree
–6	–6		
–7x	–7	x	1
x^2	1	x^2	2
$-3x^2y^2$	–3	x^2y^2	4

Division and square roots cannot be involved in the variables. The **variables** can only include **addition, subtraction and multiplication**.

Polynomials are usually written in decreasing order of terms. The largest term or the term with the highest exponent in the polynomial is usually written first. The first term in a polynomial is called the **leading term**. When the leading term contains an exponent, it tells you the degree of the term.

Here are some more examples of polynomials:

$\mathbf{6x^2 - 4xy + 2xy}$ → This three-term polynomial (also often called a trinomial) has a leading term to the second degree. It is called a second degree polynomial.

$9x^5 - 3x^4 - 2x - 2$ →This four-term polynomial has a leading term to the fifth degree and a term to the fourth degree. It is called a fifth degree polynomial.

VERY IMPORTANT:

One thing you will do when solving polynomials is combine like terms.

Like terms: 6x – 3x = 3x

Unlike terms: 6xy and 2x or 4

In this operation the first two terms are like, so they can be combined: $5x^2 + 2x^2 = 7x^2$

Exercise 5. Complete this table:

Monomial	$-7x^5$	$4x^3y^2z$	5	– 6x
Numerical Coefficient				
Degree				

Exercise 6. Complete the following table for this polynomial: $P(x) = 7x^3 - 9x - 2$

Términos	Grado	Coeficientes	Coeficiente principal	Término independiente

The numeric value of an algebraic expression is the value that we get after replacing the variable with a number and doing the operations.

Example: Calculate the numeric value of the algebraic expression **5 x + 3** , when **x = 2:**

$5 \cdot 2 + 3 = 13$

The numeric value is **13**.

Exercise 7: Calculate the numeric value of the algebraic expression for each value of *n*:

n	**1**	**3**	**7**	**10**	**15**	**20**	**30**	**50**	**100**
***3n* + 2**	**5**	**11**							

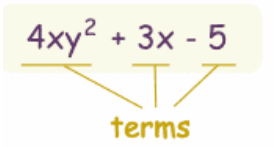
HOMEWORK.

Page 136. Exercises: 31, 32, 33, 34, 35, 36, 37.

4. OPERATIONS WITH POLYNOMIALS.

4.1. ADDING AND SUBTRACTING POLYNOMIALS.

A polynomial looks like this:



example of a polynomial
this one has 3 terms

To **add polynomials** you simply add any like terms together… so, do you remember what **like terms** are?

Like terms are terms whose variables (and their exponents such as the "2" in x^2) are the same.

In other words, terms that are "like" each other.

Note: the coefficients (the numbers you multiply by, such as "5" in 5x) can be different.

Example: $7x^2$ and x^2 and $-2x^2$ are all like terms because the variables are all x^2.

ADDING POLYNOMIALS:

Do it in two steps:

1st Place like terms together

2nd Add the like terms

Example: Add $2x^2 + 6x + 5$ and $3x^2 - 2x - 1$

Step 1: place like terms together (you can use columns for this):

$$\begin{array}{rl} & 2x^2 + 6x + 5 \\ + & 3x^2 - 2x - 1 \end{array}$$

Step 2: add the like terms: $(2+3)x^2 + (6-2)x + (5-1) = 5x^2 + 4x + 4$

Here is an animation to show you:

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html

You can add several polynomials together like that.

Example: Add (2x2 + 6y + 3xy) , (3x2 – 5xy – x) and (6xy + 5)

SUBTRACTING POLYNOMIALS

To subtract polynomials, first reverse the sign of each term you are subtracting (in other words, turn "+" into "–", and "–" into "+"), then add as usual.

You can also see an animation here:

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html

Copy the example here: _____

Example: Calculate P(x) – Q(x) where $P(x) = x^4 - 6x^3 + 7x - 8$ $Q(x) = 2x^3 - 3x^2 + 5x - 1$:

$$\begin{array}{rrrrrrrrr} x^4 & - & 6x^3 & & & + & 7x & - & 8 \\ & - & 2x^3 & + & 3x^2 & - & 5x & + & 1 \\ \hline x^4 & - & 8x^3 & + & 3x^2 & + & 2x & - & 7 \end{array}$$

4.2. MULTIPLYING POLYNOMIALS.

To multiply a polynomial:

1st Multiply each term in one polynomial by each term in the other polynomial

2nd Add all the results together, and simplify if needed.

Let's look at the simplest cases first:

1 term × 1 term (monomial times monomial)

To multiply one term by another term, first multiply the coefficients, then multiply each variable together and combine the result:

Example: Multiply the polynomials P(x) and Q(x), where:

$$P(x) = 2x^3 - 3x^2 + 5$$
$$Q(x) = x^2 - 4x + 6$$

$$\begin{array}{rrrrrrr} & & 2x^3 & - & 3x^2 & + & 5 \\ & & & & x^2 & - 4x & + 6 \\ \hline 2x^5 & - & 3x^4 & & + 5x^2 & & \\ & - & 8x^4 & + 12x^3 & & - 20x & \\ & & & 12x^3 & - 18x^2 & & + 30 \\ \hline 2x^5 & - & 11x^4 & + 24x^3 & - 13x^2 & - 20x & + 30 \end{array}$$

Other examples:

http://www.mathsisfun.com/algebra/polynomials-multiplying.html

Copy some of them here: _____

HOMEWORK.

Page 133. Exercises: 15, 16, 17, 18, 19, 20, 21.

5. REMARKABLE EXPRESSIONS. (= Igualdades notables)

5.1. ADDITION SQUARED. (= Cuadrado de una suma)

First, think about this: $(3 + 4)^2 \dots 3^2 + 4^2$

So, $(a + b)^2 \neq a^2 + b^2$.

It is very important to learn this law:

The square of an addition is equal to the sum of the square of the first term plus two times the first term multiplied by the second term, plus the square of the second term. (In Spanish: el cuadrado de una suma es igual al cuadrado del primero, más el doble del primero por el segundo, más el cuadrado del segundo):

$$\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$$

It is very easy to prove it. Try it!

Example: $(x + 5)^2 = x^2 + 10x + 25$

5.2. SUBTRACTION SQUARED. (= Cuadrado de una diferencia)

First, think about this: $(5 - 3)^2 \dots 5^2 - 3^2$

So, $(a - b)^2 \neq a^2 - b^2$

Learn this law:

The square of a subtraction is equal to the sum of the square of the first term, **minus** two times the first term multiplied by the second term, plus the square of the second term. (In Spanish: el cuadrado de una resta es igual al cuadrado del primero, más el cuadrado del segundo, **menos** el doble del primero por el segundo):

$$\mathbf{(a + b)^2 = a^2 - 2ab + b^2}$$

Example: $(x-3)^2 = x^2 - 6x + 9$ →Check it!

5.3. ADDITION TIMES SUBTRACTION. (= Suma por diferencia)

A sum multiplied by a subtraction is equal to the square of the first term **minus** the square of the second term (In Spanish: una suma por una diferencia es igual al cuadrado del primero **menos** el cuadrado del segundo:

$$\mathbf{(a+b)(a-b) = a^2 - b^2}$$

Example: $(x+7)(x-7) = x^2 - 49$ →Check it!

5.4. FACTORIAL DECOMPOSITION OF A POLYNOMIAL.

The factorial decomposition of a polynomial is an expression as a multiplication of prime factors.

(Remember the definition for numbers).

When the decomposition is easy you can do it in your mind. Look at these examples and think about them:

a) $x^2 + 5x = x(x+5)$

b) $x^2 + 8x + 16 = (x+4)^2$

c) $x^2 - 9 = (x+3)(x-3)$

d) $25x^2 - 10x + 1 = (5x-1)^2$

e) $x^2 + x + 1/4 = (x+1/2)^2$

f) $x^2 - 3 = (x+\sqrt{3})(x-\sqrt{3})$

g) $x^3 + 2x^2 + x = x(x^2+2x+1) = x(x+1)^2$

HOMEWORK.

Page 135. Exercises: 22, 23, 24, 25, 26, 27, 28.

EXERCISES

UNIT 7. POLYNOMIALS.

EXERCISE 8. Watch the following video and try to answer the following questions.

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/VRBLC/LO_Template.swf

➔ A) What is algebra?_____

_____

➔ B) What is a variable?_____

_____

EXERCISE 9. Watch the following video and then try to answer the following questions.

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/TERLC/LO_Template.swf

You have learnt some new words, let's check them:

➔ "Term" is _____
➔ "Like term" is _____
➔ "Unlike term" is "_____
➔ "Collect like terms" in Spanish is called _____"

Now, Write an example of like terms:

_____

EXERCISE 10. Do the following exercise:

http://www.bbc.co.uk/bitesize/ks3/maths/algebra/formulae1/quiz/q78311627/

Copy it in your notebook**.**

EXERCISE 11. Do the following test:

http://www.bbc.co.uk/bitesize/ks3/maths/algebra/formulae2/quiz/q51560469/

Copy it in your notebook.

EXERCISE 11. Let's revise how to expand brackets.

1. Watch the following video,

http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/BRALC/LO_Template.swf

2. Do the following operations with brackets:

a) 5 (3x – 2y +a)=_____

b) 3x (2a + 7b – 5c) = _____

EXERCISE 12. Calculate the following operations with monomials:

a) $7x^5 - 4x^5 + 9x^5$ _____ b) $-5x^2 \cdot x$ _____

c) $(-2x^5)^3$ _____ d) $-6x^3 : (-3x)$ _____

EXERCISE 13. Calculate the following operations with monomials:

a) $(3x^4)^3$ _____ b) $-5x^3 + 2x^3 + 4x^3$ _____

c) $-12x^2 : (-4x)$ _____ d) $-6x^2 \cdot (-9x) \cdot x^3$ _____

EXERCISE 14. Calculate the following operations with monomials:

a) $56x^5 : 8x$ _____ b) $6x^3 \cdot (-9x^2)$ _____

c) $-3x^2 + 15x^2 + 4x^2$____ d) $(2x^5)^2$ _____

EXERCISE 15. Calculate**:**

a) $6x^4 \cdot (-9x^3)$ _____ b) $(-3x^3)^3$ _____

c) $5x - 9x + 7x - x$ _____ d) $6x^5 : 4x$ _____

EXERCISE 16. Calculate the following multiplication with polynomials:

a) $(x^5 - 7x^3 + 6x - 1) \cdot 8x^2$ _____

b) $(2x^4 - 8x^2 + 7x - 9) \cdot 7x^3$ _____

c) $(6x^4 + 5x^3 - 8x + 7) \cdot (-9x)$ _____

d) $(x^4 - 9x^3 + 7x - 6) \cdot (-6x^4)$ _____

EXERCISE 17. Simplify the following algebraic expressions:

a) $8x - 12x^2 + 1 + 7x^2 - 3x - 5$ _____

b) $x^2 - 6x - 5x^2 + 7x^2 - 5x - 9$ _____

c) $-7x - 8 + 9x - 11x^2 + 6 + 8x^2$ _____

d) $7x^2 - 9x + 6 - 7x - 8x^2 + 12$ _____

EXERCISE 18. Expand the brackets and simplify the expressions:

a) $7x - (8x^2 + 9 + 5x^2) - 7x - 2$ _____

b) $2x^2 - 5x - 3(2x^2 + 4x^2 - 5x - 6)$ _____

c) $-(3x - 5 + 9x - 7x^2 + 4) + 10x^2$ _____

d) $7(x^2 - 6x + 9) - 7(3x - 7x^2 + 9)$ _____

EXERCISE 19. EXTRACT COMMON FACTOR:

a) $6x - 8y$ _____ b) $8x^3 - 12x^2$ _____

c) $4x^4 + 10x^3 - 6x^2$ d) $9x^2y + 6xy^2 - 3xy$ _____

_____

EXERCISE 20. Here are two polynomials:

$P(x) = 7x^3 - 4x - 1 \qquad Q(x) = -2x^2 + 5x - 3$

a) Add them: _____

_____

b) Subtract them:_____

_____

c) Multiply them:_____

_____

d) What is the degree of the addition P(x) + Q(x) ?_____
e) What is the degree of the subtraction P(x) – Q(x)?_____
f) What is the degree of the multiplication P(x)* Q(x)?_____

EXERCISE 21. Calculate in your mind:

a) $(x + 2)^0$ b) $(x - 2)^0$ c) $(x + 2)^1$ d) $(x - 2)^1$

_____ _____ _____ _____

EXERCISE 22. Calculate in your mind using the remarkable expressions:

a) $(x + 2)^2$ b) $(x - 2)^2$ c) $(x + 2)(x - 2)$

_____ _____ _____

EXERCISE 23. Calculate in your mind using the remarkable expressions:

a) $(x + 3)^2$ b) $(x - 3)^2$ c) $(x + \sqrt{3})(x - \sqrt{3})$

_____ _____ _____

EXERCISE 24. Calculate in your mind using the remarkable expressions:

a) $\left(2x + \frac{1}{2}\right)^2$ _____-

b) $\left(2x - \frac{1}{2}\right)^2$ _____

c) $\left(2x + \frac{1}{2}\right)\left(2x - \frac{1}{2}\right)$ _____

EXERCISE 25. Replace suspension points with the sign = or $\neq$:

a) $(x-3)^2 \ldots x^2 - 6x + 9$
b) $(x+2)^2 \ldots x^2 + 4$
c) $(x-3)^2 \ldots x^2 - 9$
d) $(x+2)^2 \ldots x^2 + 4x + 4$

EXERCISE 26. Extract the common factor of the following expressions:

a) $8x^2 - 12x$ _____
b) $8x^4 + 6x^2$ _____
c) $2x^4 + 4x^3 - 6x^2$ _____
d) $6x^2y + 4xy^2 - 8xy$ _____

EXERCISE 27. Here are three polynomials:

$P(x) = 7x^3 - 5x + 1$
$Q(x) = -4x^4 - 9x^2 + 4x - 7$
$R(x) = 5x^4 - 7x^3 + 5x + 6$

a) Calculate $P(x) + Q(x) + R(x)$:

b) Calculate $P(x) - Q(x) - R(x)$: