UNIT 4. MEASURES

1. TIME.

1.1. INTRODUCTION.

Time has different units of measurement - seconds, hours, days etc. Knowing how to measure time is useful because it can help you find out when your favourite TV show is on, or how many days there are in a month.

This table shows how the units we use to measure time compare.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year (366 days in a leap year)

EXERCISE 1.

a) How many minutes are there in 6 hours?

b) How many hours are there in a week?

c) How many weeks are there in 63 days?

NOTE!:

0.5 hours = 30 minutes, not 50 minutes. This is because decimals show fractions of tenths, hundredths, thousandths and so on.

But, minutes are measured in **sixtieths** of an hour. Similarly, 1/4 hour = 15 minutes, and 1/10 hour = 6 minutes.

1.2. Time - 12-hour and 24-hour clock

Time is measured using either the **12-hour clock** or the **24-hour clock**.

- **12-hour clock**

The 12-hour clock notation uses **am** and **pm** to indicate morning and afternoon.

- am is the time from 12 midnight and before 12 noon.
- pm is the time from 12 noon and before 12 midnight.

For example:

- **6.23am** means 23 minutes past 6 in the morning
- **7.45am** means quarter to 8 in the morning

- **24-hour clock**

The 24-hour clock does not require the use of am or pm. Afternoon is indicated by a number bigger than 12.

NOTE!: When converting from the 12-hour clock to the 24-hour clock remember:

for any time after 12.59 pm, add 12.

For example:

- **3.00pm** becomes (3 + 12) = 15:00
- **5.46pm** becomes (5.46 + 12) = 17:46

The 24-hour clock always uses **4 digits**, so for any time before 10:00 a zero is placed at the beginning.
For example

- 01:00 means 1.00am
- 13:00 means 1.00pm
- 04:00 means 4.00am
- 16:00 means 4.00pm

EXERCISE 2:
Copy and complete the following table, then check your answers.

12-hour clock	**24-hour clock**
6.45am	06:45
11.00pm	.....
.....	12:01
1.03am	.....
.....	15:14

1.3. Time - Timetables

The following table shows the times of the buses from Cambridge to Duxford and Whittlesford:

Cambridge	**0705**	**0805**	**0835**	**0905**
Trumpington	**0716**	**0816**	**0846**	**0916**
Great Shelford	**0722**	**0822**	**0852**	**0922**
Stapleford	**0725**	**0825**	**0855**	**0925**
Sawston	**0732**	**.....**	**0902**	**0932**
Pampisford	**.....**	**0838**	**0908**	**.....**
Duxford Wheatsheaf	**0745**	**.....**	**.....**	**0945**
Duxford Imperial War Museum	**.....**	**.....**	**0915**	**.....**
Whittlesford	**.....**	**.....**	**.....**	**0950**

EXERCISE 3.

a) I want to go to the Duxford Imperial War Museum. Which bus should I catch from Cambridge?

b) The 0805 from Cambridge is running 6 minutes late. What time does it reach Pampisford?

c) If I arrive in Sawston at 0932, what time do I leave Cambridge?

d) How long does the journey from Great Shelford to Duxford Wheatsheaf take?

1.4. Time - Calendars

Use this rhyme to help you remember how many days there are in each month:

30 days has September,
April, June and November.
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year.

EXERCISE 3. If March 28th is a Tuesday, what day is the April 6th in the same year?

1.5. Leap years

There are 365 days in a year. Therefore a leap year, with its **extra** day in February, has **366**. Leap years occur every **four** years, and are divisible by 4.

NOTE!: EXACTLY…

Our current calendar is called Gregorian and it is from 1582. It distributes a year in months, and it establishes which are leap years.

- Each ordinary year has 365 days; leap years have 366 days.
- Years which are multiples of 4 are **leap years**, except those ending in two zeros, unless they are also multiples of 400. The year 4000 and its multiples will not be leap years, either.

For example: 1996 was a leap year because 1996 ÷ 4 = 499, but

1934 was not a leap year because 1934 ÷ 4 = 483.5

EXERCISE 4. Here you have a summary about units of time.

Milenio: 1000 años.	**Mes:** 28, 29, 30 o 31 días.
Siglo: 100 años.	**Semana:** 7 días.
Década: 10 años.	**Día:** 24 horas.
Lustro: 5 años.	**Hora:** 60 minutos.
Año: 12 meses, 365 días (si es bisiesto, 366 días).	**Minuto:** 60 segundos. **Segundo:** 10 décimas de segundo.

Answer the following questions:

a) Translate the followings words into English:
Siglo:_____ Década:_____ Lustro:_____

b) How many five- year periods are there in two century?

c) What years were leap during the eighties?

d) How many seconds are there from January to July?

EXERCISE 5. Look at these examples about how to change from complex form to non-complex form measurement of time. (In Spanish: forma compleja y forma incompleja de una medida de tiempo).

***Example 1*: Convert 3 h 46 min 25 s into non-complex form. (Spanish: transforma en forma incompleja)**

$$3+\frac{46}{60}+\frac{25}{3600}=3.7736\,h$$

***Example 2:* Convert 2.7859 h into complex form. (Spanish: transforma en forma compleja)**

$$0.7859\cdot 60=47.154\,\text{min}$$

$$0.154\,\text{min}\cdot 60=9.24\,s$$

Solution: 2.7859 h = 2h 47 min 9 s

Answer these questions:

a) Calculate in your mind the non- complex form of these measurement of time:

2 h 30 min = _____ 5h 45 min = _____

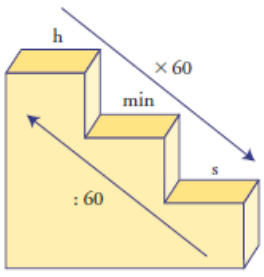
b) Convert these units of time into non-complex form:

2h 43 min 7 s = _____ 75 h 48 min 19 s = _____

c) Convert these units of time into complex form:

5.345 h = _____ 27.44 h = _____

d) Look at these example about using the calculator to convert into non-complex form:



d.1.) Convert into non-complex form using the calculator:

27 h 32 min 9 s = _____

44 h 3 min 54 s = _____

d.2.) Convert into complex form using the calculator:

15.567 h = _____

23.8 h = _____

EXERCISE 5. Calculate in your mind:

a) 2 h 20 min + 3 h 40 min

b) 7 h 45 min – 5 h 15 min

c) (2 h 15 min) × 4

d) (50 h 45 min) : 5

EXERCISE 6. After revising the example, calculate the following operations with measurements of time.

Examples:

Adding times:

2 h 42 min 35 s
+ 3 h 36 min 52 s
5 h 78 min 87 s

87 s | 60 → 27 s, 1 min

1 min

5 h 79 min 27 s

79 min | 60 → 19 min, 1 h

1 h

6 h 19 min 27 s

Subtracting times

5 h 23 min 24 s (–1 min, +60 s) → 5 h 22 min 84 s (–1 h, +60 min) → 4 h 82 min 84 s

– 3 h 32 min 41 s ⇒ – 3 h 32 min 41 s ⇒ – 3 h 32 min 41 s

1 h 50 min 43 s

5	° ’ ”	23	° ’ ”	24	° ’ ”	–	3	° ’ ”	32	° ’ ”	41	° ’ ”	=	1° 50’ 43”

Multipling times by a number

2 h 45 min 23 s

× 7

14 h 315 min 161 s

161 s | 60 → 41 s, 2 min

5 h 2 min

19 h 17 min 41 s

317 min | 60 → 17 min, 5 h

Dividing times by a number:

77 h 24 min 37 s | 5

27 15 h 28 min 55 s

2 h × 60 = 120 min

144 min

44

4 min × 60 = 240 s

277 s

27

2 s

Calculate:

a) 3 h 50 min 30 s + 6 h 42 min 37 s

b) 9 h 23 min 5 s – 5 h 52 min 16 s

c) (7 h 25 min 30 s) × 8

d) (53 h 44 min 18 s) : 6

a) Operations:

b) Operations:

c)Operations:

d) Operations:

EXERCISE 7. Calculate the following operations with measurements of time.

a) 12 h 23 min 45 s + 9 h 45 min 25 s

b) 25 h 14 min 5 s – 13 h 25 min 54 s

c) (12 h 17 min 45 s) × 9

d) (44 h 33 min 22 s) : 7

a) Operations:

b) Operations:

c) Operations:

d) Operations:

EXERCISE 8. I have attended 6 lessons, 50 minutes each. At home I have studied 2 h 30 minutes. How many hours do I have left to do other things?

EXERCISE 9. Julian went to bed at 23:02, and he woke up at 7:42. How long did he rest?

EXERCISE 10. Approximately, a year is 365.256363 days. Convert this measurement into complex form. Check it with your calculator.

EXERCISE 11. A car travels 100 metres in 9 seconds. How many metres does it travel in 1 hour?

EXERCISE 12. A bus takes 44 min 45 s on a route. If it makes 15 stops, 1 min each, how long does it take between two stops (on average)?

EXERCISE 13. To bind a magazine, 5 pupils take 20 h 30 min working together. If we pay each student € 17 per hour, how much does it cost to bind the magazine?

EXERCISE 14. From Madrid to Seville, there are 540 km. The speed of the train (AVE) is 220 km /h. How long does it take to travel the distance between the two cities?

EXERCISE 15. On Monday, Ana works 12 h 15 min and on Tuesday she works 7 h 13 min. If she earns € 7 per hour, how much does she earn on those two days?

EXERCISE 16. A phone company charges us € 3 each month. The connection cost per call is € 0.12, the first minute costs € 0.17 and each additional 30 seconds cost € 0.085.

a) **Antonio makes a phone call that lasts 55 s. How much does it cost him?**

b) **Isabel makes a phone call that lasts 2 min 25 s. How much does it cost her?**

c) Benjamín made five phone calls in November.

- **The first call lasts 42 s.**
- **The second call lasts 1 min 53 s.**
- **The third call lasts 3 min 25 s.**
- **The fourth call lasts 15 s.**
- **The fifth call lasts 44 s.**

How much does Benjamín pay in November?

d) Manuel makes two phone calls, he gets 25 % discount because he is a very good customer.

- **The first call lasts 5 min 6 s.**
- **The second call lasts 20 s.**

How much does Manuel pay for both phone calls?

2. ANGLES.

2.1. DEGREES.

We can measure angles in **degrees.**
There are 360 degrees in one full rotation (one complete circle around).
Angles can also be measured in **radians** (we'll study these later on).
The degree symbol is: °
We use a little circle ° following the number; it means degrees.

For example, 90° means 90 degrees.

Pay attention: one degree

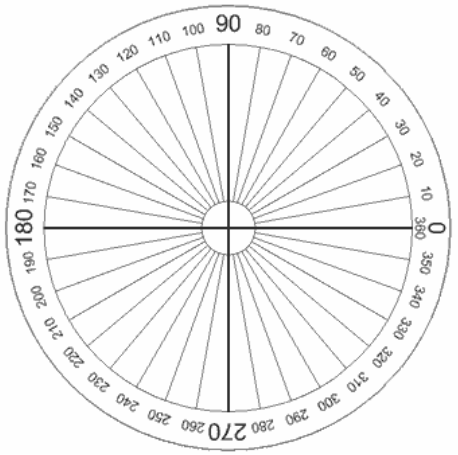
This is how large 1 degree is

The Full Circle

A Full Circle is 360°

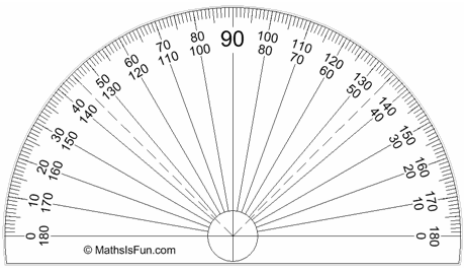
Half a circle is 180°
(called a straight angle)

A quarter of a circle is 90°
(called a right angle)



?	**Why 360 degrees?** Probably because old calendars (such as the Persian Calendar) used 360 days for a year - when they watched the stars they saw them revolve around the North Star one degree per day.

Measuring Degrees

We often measure degrees using a protractor:	You can also get full-circle protractors:
 The normal protractor measures 0° to 180°	 However, they are not as commonly used because they are a bit big and don't do anything special.

Minutes and seconds.

If we divide 1º into 60 parts with the same size, we have one minute; it is written, 1'.
If we divide 1´ into 60 parts with the same size, we have one second; it is written, 1''.

We can write an angle using complex form or non-complex form, as we studied with the measurement of time.

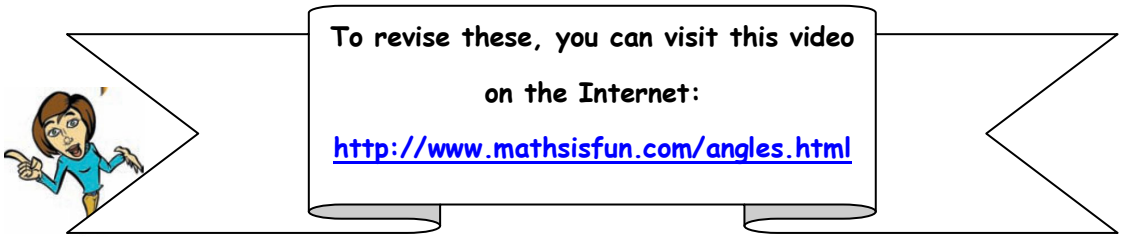
Solved examples about converting a unit into non-complex or complex form:

Complex form	Non – complex form
35 º 21' 47''	$35 + \frac{21}{60} + \frac{47}{3\,600} = 35{,}3631°$
$0{,}756° \times 60 = 45{,}36'$ $0{,}36' \times 60 = 21{,}6'' \approx 22''$ $82{,}756° = 82° 45' 22''$	82.752 º

Names of Angles

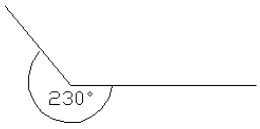
As the Angle Increases, the Name Changes

Type of Angle	Description
Acute Angle	an angle that is less than 90°
Right Angle	an angle that is 90° exactly
Obtuse Angle	an angle that is greater than 90° but less than 180°
Straight Angle	an angle that is 180° exactly
Reflex Angle	an angle that is greater than 180°



Note: Be Careful What You Measure

This is an Obtuse Angle.



And this is a Reflex Angle.

But the lines are the same... so when naming the angles make sure that you know **which angle you need to measure**!

Parts of an angle:

Did you know…?

The plural of "vertex" is "vertices".

Exercise 1. Estimate these angles using the protractor.

a)

b)

Exercise 2. Draw these angles, approximately:

a) 45º b) 90 º c) 270 º

Exercise 3. In your mind, convert these units of angles into non-complex form.

a) 18º 15' = b) 43º 30' = c) 10º 45' =

Exercise 4. In your mind, convert these units of angles into complex form.

a) 57.5º = b) 125.75º = c) 2.25º =

Exercise 5. Convert into non-complex form:

a) 23° 47' 15" b) 55° 25' 48"

Operations: Operations:

Exercise 6. Convert into complex form:

a) 41,1234° b) 83,67°

Operations: Operations:

Exercise 7. Using the calculator, convert into non-complex form:

a) 44° 25' 36" b) 77° 53' 41"

Exercise 8. Using the calculator, convert into non-complex form:

a) 58,506° b) 144,87°

Exercise 9. In your mind, calculate these operations with units of angles:

a) 25° 30' + 40° 30' b) 57° 45' – 47° 15'

c) (10° 15') × 4 d) (60° 30') : 3

Exercise 10. Calculate these operations with units of angles (Use exercise 7 on page number 7 to revise the procedure):

a) $25^\circ\ 30' + 40^\circ\ 30'$

b) $57^\circ\ 45' - 47^\circ\ 15'$

c) $(10^\circ\ 15') \times 4$

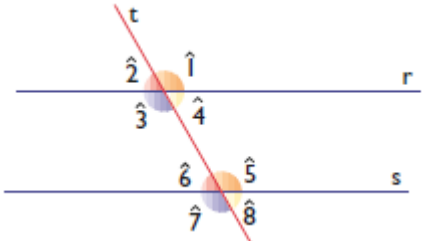
d) $(60^\circ\ 30') : 3$

Exercise 11. A right angle of a right triangle measures 23º 44' 53'' . How long is the others angles of the triangle?

Exercise 12. Calculate the angles of a equilateral triangle, if we know all together add 180º .

Exercise 13. If an angle measures 43º 28' 45'', calculate the complementary angle.

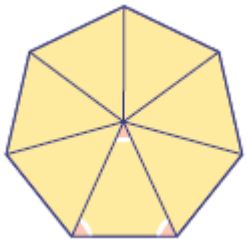
Exercise 14. If angle $\hat{1}$ measures 119º 12' 37'' . Calculate the others angles in the following drawing:



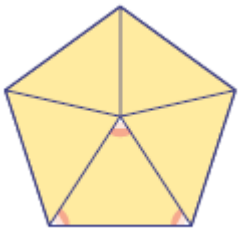
Exercise 15. If we divide a right angle into 7 parts with the same size, how long is each part?

Exercise 16. In an isosceles triangle there are two angles which measure 45º 55' 17''. How long is the other angle?

Exercise 17. Calculate the angles in this regular heptagon:



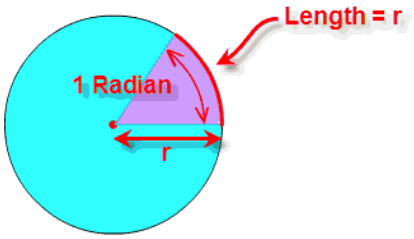
Exercise 18. Calculate the angles in this regular pentagon:



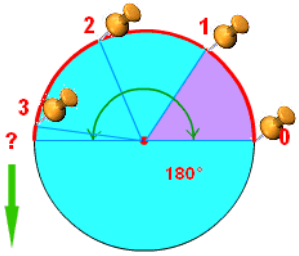
2.2. Radians

Angles can be measured in **degrees** or **radians**.

One radian is 180/π degrees, or about 57.296°



A radian is the **angle** made by taking the **radius** and **wrapping it along the edge** of the circle:



? = 3.14159... = π

A half rotation = 180° = π Radians

1 Radian = 180°/ π = 57.2958...°

To understand better what a radian is, you can watch this videoclip on the Internet:
http://www.mathsisfun.com/geometry/radians.html

There are 2π radians in a full circle.

In other words, if you cut up pieces of string exactly the length from the **center of a circle to its edge**, how many pieces would you need to go **around the edge** of the circle?

Answer: 2**π**, or about 6.28 pieces of string.

Radians are preferred by mathematicians

Because the radian is based on the pure idea of *"the radius being laid along the circumference*", it gives simple and natural results to many angle-related mathematics.

Conclusion

So, degrees are easier to use in everyday work, but radians are much better for high-level mathematics.

Solved examples:

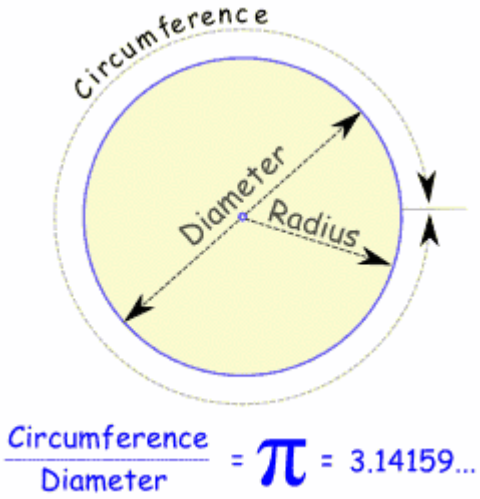
Convert into radians the following angles (remember **2**π = 360º = full circle):

180º = $\frac{2\Pi}{2} = \Pi$ **radians**

45º = $\frac{\Pi}{2} : 2 = \frac{\Pi}{4}$ **radians**

90º = $\frac{2\Pi}{4} = \frac{\Pi}{2}$ **radians**

But … do you know what is the number PI? /paɪ/

π	**Pi** (the symbol is the Greek letter π) is: *The ratio of the **Circumference** to the **Diameter** of a Circle.*
In other words, if you measure the circumference, and then divide it by the diameter of the circle, you get the number π. It is approximately equal to: **3.14159265358979323846…** The digits go on and on with no pattern. In fact, pi has been calculated to over one million decimal places, and still there is no pattern.	

Exercise 19. Draw the following angles approximately and convert them into radians, in your mind:

a) 30 º

b) 60 º

c)180 º

d) 150 º

e) 270 º

f) 135º

Exercise 20. Draw the following angles approximately and convert them into degrees, in your mind.

a) $\frac{\Pi}{2} =$

b) $\Pi =$

c) $\frac{\Pi}{6} =$

d) $3 \cdot \frac{\Pi}{2} =$

e) $5 \cdot \frac{\Pi}{2} =$

f) $3 \cdot \frac{\Pi}{4} =$

INTERESENTING LINKS ABOUT ANGLES.

- Concepto de ángulo, tipos, consecutivos y adyacentes http://www.aplicaciones.info/decimales/geoele03.htm
- Medir ángulos, complementarios, suplementarios, etc http://www.aplicaciones.info/decimales/geoele04.htm
  http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_and_space/angles_1_4.shtml
- Ángulos complementarios y suplementarios en inglés http://www.mathsisfun.com/geometry/complementary-angles.html
  http://www.mathsisfun.com/geometry/supplementary-angles.html
- Medir con el transportador (animado) y un ejercicio de medir http://www.mathsisfun.com/geometry/protractor-using.html
  http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html
- Ejercicios de medir ángulos http://www.rainforestmaths.com/ (elegir grado 6, 2D-shapes, angles)
- Estimar ángulos: http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html
- Vídeo sobre los tipos de ángulos en inglés http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/ANTLC/launch.html
- Tipos de ángulos en inglés. Juego de parejas http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_and_space/angles_1_2.shtml
- ¿Cómo dibujar ángulos? http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_and_space/angles_1_3.shtml
- Test sobre medida de ángulos http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_and_space/angles_1_test.shtml
- Ángulos en una línea http://www.mathsisfun.com/angle180.html
- Ángulos en un punto http://www.mathsisfun.com/angle360.html
- Ángulos que faltan http://www.thatquiz.org/tq/practice.html?angles
- Vídeo de como trazar la mediatriz de un segmento http://lgfl.skoool.co.uk/content/keystage3/maths/pc/learningsteps/BILLC/launch.html
- Bisectriz de un segmento http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_and_space/construction_4.shtml
- Un poco de repaso y test: http://www.bbc.co.uk/schools/ks2bitesize/maths/revision_bites/angles1.shtml
  http://www.bbc.co.uk/schools/ks2bitesize/maths/tests/angles.shtml
- Diccionario interactivo: http://www.teachers.ash.org.au/jeather/maths/dictionary.html
- Juego sobre ángulos http://www.hittingthetarget.com/hittingthetarget.php
- Juego de estimar ángulos http://www.mathplayground.com/alienangles.html