

UNIT 2. FRACTIONS AND DECIMAL NUMBERS

1. FRACTIONS.

1.1. WHAT ARE FRACTIONS?

Fractions are numbers for counting PART of something.

A **fraction** is written as two whole numbers, one above the other, separated by a horizontal line.

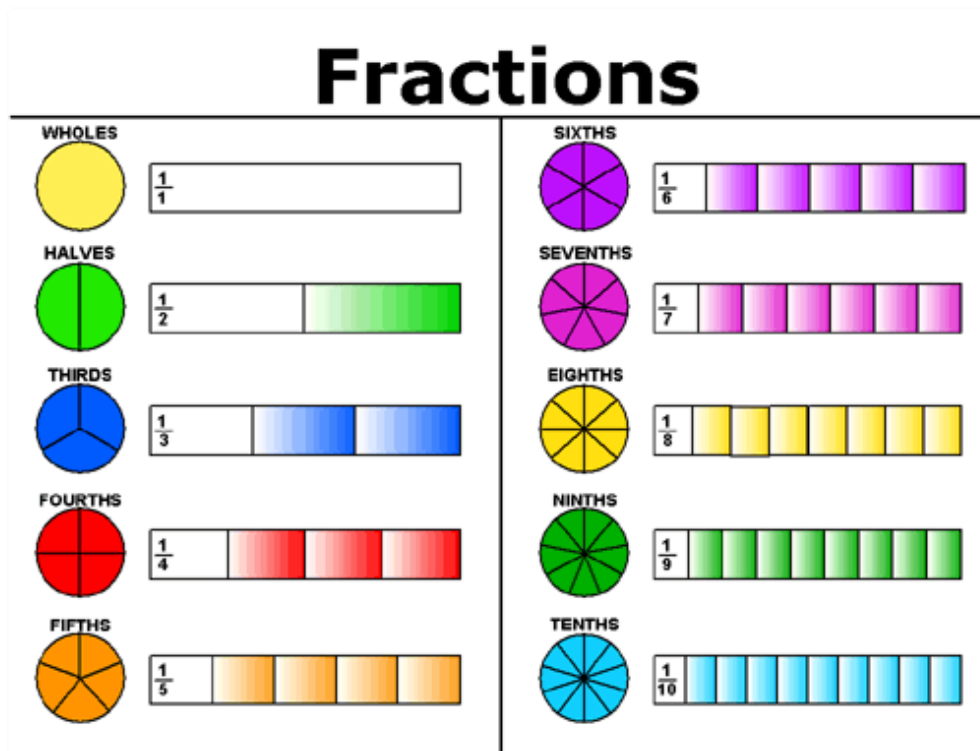
$$\frac{1}{6}$$

← The top number is called the numerator.

← The bottom number is called the denominator.

We read this as "one sixth."

A fraction is a part of an entire object.



- Fraction as an operator:** to calculate a fraction of a number, divide the number by the denominator and multiply the result by the numerator.

$$\frac{3}{5} \text{ of } 30 = 30 \div 5 \cdot 3 = 6 \cdot 3 = 18$$

- Types of fractions:

Proper fraction: the numerator is smaller than the denominator; it is less than 1, for example $\frac{1}{2}$.

Improper fraction: the numerator is bigger than the denominator; it is greater than 1, for example $\frac{5}{3}$.

- Equivalent Fractions:

As you know, $\frac{1}{2}$ is the same amount as $\frac{3}{6}$, so

$$\frac{1}{2} = \frac{3}{6}$$

These are called equivalent fractions. (Because they are equal amounts.)

Example: Is $\frac{3}{5} = \frac{39}{65}$?

Here's a quick trick to see if two fractions are equivalent; we call this trick cross-multiplication:

We get the same result on both sides, so... $\frac{3}{5} = \frac{39}{65}$!

- Simplifying Fractions:

Example: Let's reduce $\frac{27}{108}$. First, let's divide it! $\frac{27 \div 3}{108 \div 3} = \frac{9}{36}$

Hey, now it's easy! Do you see that 9 goes in?

$\frac{9 \div 9}{36 \div 9} = \frac{1}{4}$ so $\frac{27}{108} = \frac{1}{4}$. This is the lowest term.

- Inverse fraction:

The inverse fraction of another fraction is the one where the denominator exchanges places with the numerator with the same sign.



Example: The inverse fraction of $\frac{3}{4}$ is $\frac{4}{3}$.

The inverse fraction of $-\frac{2}{5}$ is

- **Opposite fraction:**

The opposite fraction of another fraction is the one where the sign is changed.

Example: The opposite fraction of $\frac{2}{3}$ is .

The opposite fraction of $-\frac{2}{5}$ is .

1.2. OPERATIONS WITH FRACTIONS.

Let's revise how to do operations with fractions.

1.2.1. ADD AND SUBTRACT FRACTIONS.

When the denominators are the same. Easy! Try it alone! Write one example!

When the denominators are different.

Let's try this: $\frac{1}{2} + \frac{1}{3}$

The main rule of this game is that we can't do anything until the denominators are the same!

We need to find something called the least common denominator (LCD)... It's really just the LCM of our denominators, 2 and 3.

The LCM of 2 and 3 is 6. So, our LCD is 6.

We need to make this our new denominator...

Change the $\frac{1}{2}$: $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

Change the $\frac{1}{3}$: $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$



So $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$ Now we can do it! $\frac{3+2}{6} = \frac{5}{6}$

Summing up:

How to add or subtract fractions with different denominators:

- 1°. Find the Least Common Denominator (LCD) of the fractions
- 2°. Rename the fractions to have the LCD
- 3°. Add or subtract the numerators of the fractions
- 4°. Simplify the Fraction

1.2.2. How to Multiply Fractions.

This is easy!

We just multiply straight across...

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

Then just reduce it

1.2.3. How to Divide Fractions.

This is easy too, we just do cross-multiplication...

$$\frac{1}{3} \div \frac{4}{5} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$$

1.2.4. Powers of Fractions.

Simply multiply the numerator and the denominator by themselves as many times as the power (exponent) indicates.

$$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$$



1.2.5. Order of operations.

The order of operations for fractions is the same as that for natural numbers or integers.

Remember: BODMAS.

B = Brackets.

O = Orders (Exponents or indexes)

DM = Division and Multiplication.

AS = Addition and Subtraction.

If you want to learn this rule with a song, you can visit this video:

<http://www.youtube.com/watch?v=TMTESTKAjPg>

EXERCISES

1. Calculate:

a) $\frac{2}{3}$ of 18 =

b) $\frac{4}{7}$ of 35 =

c) $\frac{3}{8}$ of 80 =

d) $\frac{7}{5}$ of 135 =

2. Compare the following fractions: $\frac{2}{3}$, $\frac{23}{4}$, $\frac{5}{5}$. Which is the biggest? Which is the smallest?

3. Draw the following numbers on the real line: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{3}$, $\frac{11}{4}$, $\frac{7}{2}$, $\frac{14}{3}$.

4. Calculate in your mind:

a) $\frac{1}{2} + 1$

b) $2 - \frac{1}{3}$

c) $2 \cdot \frac{3}{5}$

5. Calculate the inverse fraction and the opposite fraction of:

$\frac{2}{3}$, $-\frac{4}{5}$, $\frac{1}{2}$, $-\frac{1}{3}$



6. Work out:

a) $\frac{1}{3} - \frac{5}{6} + \frac{3}{2}$

b) $\frac{7}{9} + \frac{11}{12} - \frac{5}{6}$

c) $\frac{3}{8} - 2 + \frac{5}{6}$

d) $\frac{2}{35} + \frac{8}{7} - \frac{3}{10}$

7. Work out:

a) $\frac{2}{9} \cdot \frac{15}{4}$

b) $5 \cdot \frac{3}{25}$

c) $\frac{7}{12} : \frac{3}{4}$

8. Work out:

a) $3 - \left(\frac{1}{4} + \frac{5}{3} \right)$

b) $1 - \left(\frac{3}{2} - \frac{7}{5} \right)$

c) $\frac{2}{3} \cdot \left(\frac{1}{3} + \frac{4}{9} \right)$

d) $\left(\frac{1}{2} - \frac{3}{8} \right) : \frac{3}{4}$

9. 40 students joined the soccer club. $\frac{5}{8}$ of the students were boys. How many girls joined the soccer club?



10. Kelly had \$24. She spent $\frac{5}{6}$ of her money on a book. How much money did Kelly have left?
11. Anna bought some stickers. She gave 9 stickers to her sister. Anna then had $\frac{3}{4}$ of her stickers left. How many stickers did Anna buy?
12. Danny spent $\frac{1}{5}$ of his money on comic books. The comic books cost \$7 altogether. How much money did Danny have left?
13. Lori used $\frac{1}{4}$ of her rubberbands in a project. She had 27 rubberbands left. How many rubberbands did Lori use in her project?
14. Marcus had \$450.
He spent $\frac{2}{5}$ of it on a DVD player.
Then he spent $\frac{1}{3}$ of the remaining money on a jacket.
How much money did Marcus have left?



15. The teen club had a cupcake sale.
They sold $\frac{2}{3}$ of their cupcakes in the morning.
They sold $\frac{1}{6}$ of their cupcakes in the afternoon.
They sold 200 cupcakes altogether.
How many cupcakes were left to sell?
16. Chef Pillsbury bought some eggs.
He used $\frac{1}{2}$ of them to make some pies.
He used $\frac{1}{4}$ of the remaining eggs to make a cake.
He then had 15 eggs left.
How many eggs did Chef Pillsbury buy?
17. Leanne bought a bag of marbles.
 $\frac{3}{8}$ of the marbles were green.
 $\frac{1}{5}$ of the remaining marbles were red.
If there were 6 red marbles, how many marbles were in the bag altogether?
18. Kara read 20 pages of her new book on Saturday.
She read $\frac{1}{4}$ of the remaining pages on Sunday.
She still has 36 pages to read.
How many pages are there in Kara's book?



19. Calculate.

a) $\left(\frac{2}{5} - 1\right) \cdot \frac{5}{3}$

b) $\left(\frac{4}{5} - 2\right) \cdot \left(\frac{7}{4} - \frac{5}{6}\right)$

c) $\left(\frac{4}{3} - 2\right) \cdot \frac{3}{10}$

d) $\left(2 - \frac{7}{6}\right) \cdot \left(4 - \frac{7}{3}\right)$

Solutions: a) -1 b) -11/10 c) -1 / 5 d) 25 /18

20. Work out:

a) $\frac{1}{4} + 5 - \left(\frac{5}{8} + \frac{3}{2}\right)$

b) $3 - \left(\frac{5}{6} - 2\right) + \frac{2}{3}$

c) $\frac{1}{2} : \left(\frac{2}{5} - \frac{3}{10}\right)$

d) $3 - \left(\frac{5}{2} - \frac{7}{4}\right) : \frac{3}{2}$

Solutions: a) 25/8 b) 29/6 c) 5 d) 5/2



**FOR MORE PRACTICE
EXERCISES YOU CAN DO ON
THE INTERNET, VISIT THIS
USEFUL AND ENJOYABLE
WEBSITE. COME ON! TRY
IT!**



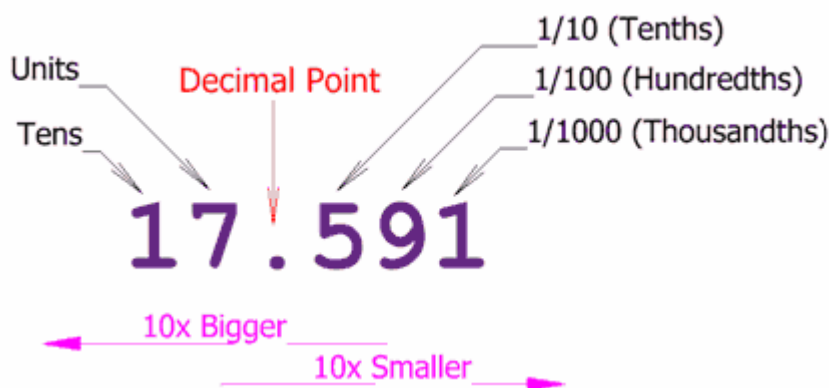
<http://www.mathplayground.com/wpdatabase/wpindex.html>

2. DECIMAL NUMBERS.

2.1. DECIMAL POINT

The **decimal point** is the most important part of a Decimal Number. It is exactly to the right of the Units position. Without it, we cannot know what each position means.

Now we can continue with smaller and smaller values, from **tenths**, to **hundredths**, and so on, as in this example:



Large and Small

So, our Decimal System lets us write numbers as large or as small as we want to, using the decimal point. Numbers can be placed to the left or to the right of a decimal point, to indicate values greater than one or less than one.

17.591



The number to the left of the decimal point is a whole number (17 for example)

As we move further left, every number place gets **10 times bigger**.



The first digit on the right means **tenths** (1/10).

As we move further right, every number place gets **10 times smaller** (one tenth as big).



2.2.DEFINITION OF DECIMAL



The word "Decimal" really means "based on 10" (From Latin *decima: a tenth part*).

We sometimes say "decimal" when we refer to anything related to our numbering system, but a "Decimal Number" usually means "a number which has a Decimal Point".

Ways to think about Decimal Numbers ...

➤ ... as a Whole Number Plus Tenths, Hundredths, etc

You can think of a decimal number as a whole number plus tenths, hundredths, etc:

Example 1: What is 2.3 ?

- On the left side is "2", that is the whole number part.
- The 3 is in the "tenths" position, meaning "3 tenths", or 3/10
- So, 2.3 is "2 and 3 tenths"

Example 2: What is 13.76 ?

- On the left side is "13", that is the whole number part.
- There are two digits on the right side, the 7 is in the "tenths" position, and the 6 is the "hundredths" position
- So, 13.76 is "13 and 7 tenths and 6 hundredths"

➤ ... as a Decimal Fraction.

Or, you can think of a decimal number as a Decimal Fraction.

A Decimal Fraction is a fraction where the denominator (the bottom number) is a number such as 10, 100, 1000, etc (in other words a power of ten)

So "2.3" can look like this: $\frac{23}{10}$

And "13.76" can look like this: $\frac{1376}{1000}$

➤ .. as a Whole Number and Decimal Fraction

Or, you can think of a decimal number as a Whole Number plus a Decimal Fraction.



So "2.3" can look like this: 2 and $\frac{3}{10}$

And "13.76" would look like this: 13 and $\frac{76}{100}$

Those are all correct ways to think of decimal numbers.

2.3. Rounding Numbers

Rounding means reducing the digits in a number while trying to keep its value similar. The result is less accurate, but easier to use.

Example: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

There are several **different methods for rounding**, but here we will only look at the common method, the one used by most people ...

How to Round Numbers

Decide which is the last digit to keep.

Increase it by 1 if the next digit is 5 or more (this is called rounding up).

Leave it the same if the next digit is less than 5 (this is called rounding down).

In other words, if the first digit removed is 5 or more, then increase the last digit remaining by 1.



NOTE: Why does 5 go up ? *Think about sport ... you should have the same number of players on each team, right?*



- 0,1,2,3 and 4 are on team "down"
- 5,6,7,8 and 9 are on team "up"

(This is the most important part of the "common" method of rounding)

➤ Rounding Decimals

To understand it, check these examples:



Examples	Because ...
3.1416 rounded to hundredths is 3.14	... the next digit (1) is less than 5
1.2635 rounded to tenths is 1.3	... the next digit (6) is 5 or more
1.2635 rounded to 3 decimal places is 1.264	... the next digit (5) is 5 or more

➤ **Rounding Whole Numbers**

You may want to round to tens, hundreds, etc, In this case you replace the removed digits with zero.

Examples	Because ...
134.9 rounded to tens is 130	... the next digit (4) is less than 5
12,690 rounded to thousands is 13,000	... the next digit (6) is 5 or more
1.239 rounded to units is 1	... the next digit (2) is less than 5

➤ **Rounding to Significant Digits**

To round "so many" significant digits, just count that many from left to right, and then round off from there. (Note: if there are leading zeros (such as 0.006), don't count them because they are only there to show how small the number is).

Examples	Because ...
1.239 rounded to 3 significant digits is 1.24	... the next digit (9) is 5 or more
134.9 rounded to 1 significant digit is 100	... the next digit (3) is less than 5
0.0165 rounded to 2 significant digits is 0.017	... the next digit (5) is 5 or more

2.4. Ordering Decimals

Ordering decimals can be tricky. This is because often we look at 0.42 and 0.402 and say that 0.402 must be bigger because there are more digits.

Example

Order the following decimals:

0.402, 0.42, 0.375, 1.2, 0.85

TO PRACTISE MORE EXERCISES ABOUT ORDERING NUMBERS, YOU CAN VISIT THIS WEBSITE:

<http://www.mathsisfun.com/numbers/ordering-game.php>

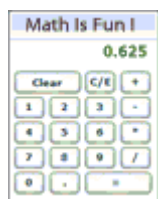
AND PLAY THIS GAME:

<http://www.mathsisfun.com/numbers/ordering-game.php?m=Dec-Tricky>



2.5. Convert Fractions to Decimals

The simplest method is to use a calculator.



Just divide the top of the fraction by the bottom, and read off the answer !

Example: What is $\frac{5}{8}$ as a fraction?

... get your calculator and type in "5 / 8 =", the answer should be 0.625

But, of course, you can divide manually.

2.6. Convert Decimals to Fractions

➤ IF THE NUMBER IS AN EXACT DECIMAL:

To convert a Decimal to a Fraction follow these steps:

Step 1: Write in the numerator, the number without the decimal point.

Step 2: Write in the denominator, 10 if the number has one decimal number, 100 if it has two decimal numbers, 1000 if it has three decimal numbers, etc.

Step 3: Simplify (or reduce) the fraction.

Example: Express 0.75 as a fraction:

$$\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$$

Example: Express 1.225 as a fraction:

$$\frac{1225}{1000} = \frac{245}{200} = \frac{49}{40}$$

➤ IF THE NUMBER IS A RECURRING DECIMAL: (NÚMERO DECIMAL PERIÓDICO)

1st case: Periódico puro.

$$\text{Example: } 1.3434... = 1.34 = \frac{133-1}{99} = \frac{132}{99} = \frac{44}{33} = \frac{4}{3}$$

2nd case: Periódico mixto.

$$\text{Example: } 5.127373... = 5.1273 = \frac{51273-512}{9900} = \frac{50761}{9900}$$



2.7. Rounding

Significant figures.

We do not always need to give detailed answers to problems – sometimes we just want a rough idea. When we are faced with a long number, we can round it off to the nearest thousand, or to the nearest million. And when we get a long decimal answer on a calculator, we can round it off to a certain number of decimal places.

Another method of giving an approximate answer is to round off using significant figures. Sometimes the term 'significant figures' is abbreviated to sig. figs. and often it's abbreviated to just s.f.

The word «significant» means «important». The closer a digit is to the beginning of a number, the more important -or significant- it is.

In number 368249, 3 is the most significant digit, because it tells us that the number is 3 hundred thousand and something. Therefore, 6 is the next most significant, and so on.

With number 0.0000058763, 5 is the most significant digit, because it tells us that the number is 5 millionths and something. 8 is the next most significant, and so on.

We round off a number using a certain number of significant figures. The most common are 1, 2 or 3 significant figures.

Remember: the rules for rounding up or down are the same as before:

If the next number is 5 or more, we round up.

If the next number is 4 or less, we do not round up.

Question 1:

What do you get if you write number 368249 rounded to 1 significant figure?

Question 2:

What do you get if you write number 0.00245 rounded to 1 significant figure?

Question 3:

What do you get if you write 0.0000058763 rounded to 2 significant figures?

2.8. ESTIMATING

We can use significant figures to get an approximate answer (that is, an estimate) to a problem.



We can round off all the numbers in a maths problem to 1 significant figure to make 'easier' numbers. It is often possible to do this in your head.

SOLVED Question 4:

Find an estimate for $\frac{19.4}{0.0437}$

SOLUTION:

We first round off both numbers to 1 significant figure (s.f.):

$$19.4 = 20 \text{ (1 s.f.)}$$

$$0.0437 = 0.04 \text{ (1 s.f.)}$$

Now we need to make the denominator a whole number. We can do this by multiplying both 20 and 0.04 by 100.

$$\frac{20}{0.04} = \frac{20 \times 100}{0.04 \times 100} = \frac{2000}{4}$$

Next, we divide everything by 4.

$$= \frac{2000}{4} = 500$$

The real answer to $19.4 \div 0.0437 = 443.9359...$ So 500 was a good estimate.

Question 5:

How can you get an estimate for 386062×0.007243 ?

EXERCISES:

1. Round off 38379 to the nearest hundred.
2. Round off 38379 to the nearest hundred.



3. Round off 38379 to the nearest hundred.
4. Round off the number 37649 to 1 s.f.
5. What sum would you do to get an estimate for 73541×0.008564 (to 1 s.f.)?
6. Calculate $25 \div 4.7$ to 1 decimal place (d.p.).
7. Calculate 4.77×2.4 to 2 d.p.
8. Calculate $1399 \div 199.99$ to 2 d.p.

2.9. OPERATIONS WITH DECIMAL NUMBERS

To revise operations with decimal numbers you can visit this website:

<http://www.bbc.co.uk/schools/ks3bitesize/maths/number/decimals/activity.shtml>

EXERCISES:

1. Which is the biggest?
A) 0.75×10
B) 0.7×100
C) 0.8×10
2. A booklet contains 100 pieces of paper, and it is 13mm thick. What is the thickness of one sheet of paper?
3. £8.40 is shared equally between three brothers. How much does each brother receive?



4. Aisha buys a magazine for £1.25 and chocolates for £2.99. She pays with a £10 note. How much change does she receive?
5. Jolene loves baking. She is baking cookies for the community bake sale this weekend. She has made bags of six cookies that she intends to sell for \$3.00. Jolene sold 19 bags of Christmas-tree-shaped cookies and 18 bags of bell-shaped cookies.
How much did Jolene earn from Christmas-tree-shaped cookies? _____
How much did Jolene earn from bell-shaped cookies? _____
How much money did Jolene earn in total? _____
6. Page 44, exercise 110.
7. Page 44, exercise 111.
8. Page 44, exercise 112.



TO REVISE FOR THE TEST.

EXERCISE 1. Calculate:

a) $2 - \frac{5}{6} + \frac{1}{4}$

b) $\frac{2}{3} - \left(\frac{1}{2} + \frac{3}{4}\right)$

Answer: a) $\frac{17}{12}$, b) $-\frac{7}{12}$

EXERCISE 2. Calculate:

a) $\left(\frac{4}{5} - 2\right) \cdot \left(\frac{7}{4} - \frac{5}{6}\right)$

b) $\frac{7}{5} : \frac{7}{10} - \frac{5}{6} : \frac{1}{4}$

Answer: a) $-\frac{11}{10}$ b) $-\frac{4}{3}$

EXERCISE 3. a) Calculate with TWO decimal numbers. 42,7 : 7,08

b) $(45,14 - 13,205) \cdot 9,6$

Answer: a) 6,03 b) 306,576

EXERCISE 4. The perimeter of an equilateral triangle is 24.8 m long. How many metres long is a side? Round to centimetres.

Answer: 8,27 m = 827 cm

EXERCISE 5. Write the following decimal numbers as a fraction:

a) 1,25

b) $8,\overline{3}$

c) $2,\overline{681}$

Answer: a) $\frac{125}{100} = \frac{5}{4}$ b) $\frac{25}{3}$ c) $\frac{59}{22}$

EXERCISE 6. A car uses 31.32 litres of petrol in 540 km. How many litres does it use in 100 km?

Answer: 5,8 litros.



EXERCISE 7. Marta used $\frac{3}{5}$ of her money to buy some CD's, and $\frac{1}{2}$ of the remaining money to buy a gift for her sister.

- a) Which fraction of her money does she spend?
- b) If she has € 6 left, how much did she have in the beginning?

Answer: a) She spent: $\frac{4}{5}$ b) She had €30 in the beginning.

