

UNIT 1.DIVISIBILITY AND INTEGERS

1.MULTIPLES AND FACTORS

1.1.Concept of multiple

We say that a number "a" is a **multiple** of another number "b" if the division $a : b$ is an **exact division**, that is, if "b" contains "a" a whole number of times.

(Para obtener los múltiplos de un número lo multiplicamos por 1, 2, 3 y así sucesivamente.)

Solved Example:

Obtain some multiples of 3, 5 and 7:

$3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, 3 \times 5, 3 \times 6 \dots$ so Multiples of 3 are 3, 6, 9, 12, 15, 18,

$5 \times 1, 5 \times 2, 5 \times 3, 5 \times 4, 5 \times 5, 5 \times 6 \dots$ so Multiples of 5 are 5, 10, 15, 20, 25, 30,

$7 \times 1, 7 \times 2, 7 \times 3, 7 \times 4, 7 \times 5, 7 \times 6 \dots$ so Multiples of 7 are 7, 14, 21, 28, 35, 42,

1.2. Concept of factor

We say that a number "b" is a **factor** of another number "a" if the division $a : b$ is an exact division.

In Spanish:

Por tanto, si la división $a : b$ es exacta, entonces a (el número más grande) es el múltiplo y b (el número más pequeño) es el divisor.

(Para encontrar los divisores de un número debemos probar a dividirlo por todos los números naturales que son más pequeños que él. Pero hay un pequeño truco que es irlos agrupando por parejas de divisores: Empezamos dividiendo por 1, 2, 3... y si encontramos un divisor, el cociente es otro divisor. Seguimos así hasta que empiecen a repetirse).

Solved Example:

Obtain all the factors of 90:

$$\begin{array}{llllll} 90 : 1 = 90; & 90 : 2 = 45; & 90 : 3 = 30; & 90 : 4 = \text{not possible}; & 90 : 5 = 18; & 90 : 6 = 15; \\ 90 : 7 = \text{n.p.}; & 90 : 8 = \text{n.p.}; & 90 : 9 = 10; & & & \\ 90 : 10 = 9 & & & & & \end{array}$$

(10 and 9 are repeated, so we have finished)

So, the factors of 90 are: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

Solve the following exercises:

1. Find three multiples of 11 that are between 27 and 90.

2. Work out if 556 is a multiple of 4.

3. Find out if 12 is a factor of 144.

4. Which of these numbers is a factor of 91?
a) 3 b) 7 c) 11 d) 13

5. Work out all the factors of the following numbers:
a) 24
b) 27
c) 48
d) 25
e) 7
f) 56

6. Point out which of these numbers have exactly three factors.
a) 4
b) 25
c) 15
d) 49



1.3. The properties of multiples and factors

Multiples	Factors
a) Every number is the multiple of itself <i>Cada número es múltiplo de sí mismo</i> Example: 3 is the multiple of 3	a) Every number is the factor of itself <i>Todo número es divisor de sí mismo</i> Example: 3 is the factor of 3
b) Every number is the multiple of 1: <i>Todos los números son múltiplos de 1</i> Example: 7 is the multiple of 1 c) Zero is the multiple of any number <i>cero es múltiplo de cualquier número</i> Example: 0 is the multiple of 3 d) Every number has an infinite number of multiples. <i>Todos los números tienen infinitos múltiplos</i>	b) 1 is the factor of any number <i>1 es divisor de cualquier número</i> Example: 1 is the factor of 7 c) Zero is not the factor of any number <i>Cero no es divisor de ningún número</i> Example: Zero is not the factor of 2 d) The set of the factors of a number is finite <i>El conjunto de divisores de un número es finito</i>

2. PRIME AND COMPOSITE NUMBERS

So, a **prime number** only has two factors: the number one and itself. For example: 3, 5, 11, 17, etc.

A **composite number** has more than two factors. For example: 4, 9, 15, 30, etc.

Para averiguar si un número es primo o compuesto puedes hallar sus divisores, o bien dividirlo por todos los números primos menores que él, Si no encuentras ningún divisor, entonces el número es primo.

A smart procedure to find the first prime numbers is the Sieve of Erathostenes. It consists of a table with the numbers from 1 to 100, like the one below, and now follow the following rules:

- Number 2 is prime. Circle it, then cross out all the multiples of 2
- Circle the next number that is not crossed out (3) because it is prime too. And then, cross out all its multiples.
- Continue in this way, that is, circle the numbers which are not crossed out and cross out all their multiples until you finish with the table. Then you will have the first prime numbers under 100.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solve the following exercise:

Work out the factors of the numbers below and then point out which ones are prime numbers:

a) 8

b) 101

c) 57

d) 49

3.- DIVISIBILITY RULES

These rules help you to know if a number is a multiple of another without doing the division:

In Spanish: las reglas de divisibilidad te ayudan a saber si un número es múltiplo de otro sin hacer la división:

● Rule of number 2: A number is divisible by 2 if its last digit is either 0 or an even number. Un número es divisible por 2 si su última cifra es 0 ó un número par. Example: 46, 200, 34, 108.....

● Rule of number 3: A number is divisible by 3 if the sum of its digits is a multiple of 3. Un número es divisible por 3 si la suma de sus cifras es múltiplo de 3. Example: 45, 105, 300, 417....

● Rule of number 4: A number is divisible by 4 if its two last digits are multiples of 4. Un número es divisible por 4 si sus dos últimas cifras son múltiplo de 4. Example: 100, 224, 340, 664....

● Rule of number 5: A number is divisible by 5 if it ends in 0 or 5. Un número es múltiplo de 5 si acaba en 0 ó 5. Example: 200, 345, 650, 800

● Rule of number 9: A number is divisible by 9 if the sum of its digits is a multiple of 9.



Un número es divisible por 9 si la suma de sus cifras es múltiplo de 9. Example: 81, 333, 450, 1278.....

● Rule of number 10: A number is divisible by 10 if it ends in 0. Un número es divisible por 10 si acaba en 0. Example: 30, 400, 500.

● Rule of number 11: A number is divisible by 11 if the difference between the sum of the digits on odd positions and the sum of the digits on even positions is 0, 11 or a multiple of 11. Un número es divisible por 11 si la diferencia entre la suma de las cifras en posición par y la suma de las cifras en posición impar es 0, 11 o un múltiplo de 11. Example: 121, 3652

Solve the following exercises:

1. Use the divisibility rules to complete the following table:

Divisible by	2	3	4	5	9	10	11	25	100
375									
990									
1.848									
12.300									
14.240									

2. Find out two numbers with five digits that are divisible by both 2 and 5 and aren't divisible by 100.
3. Write down two numbers with five digits that are multiples of:
 - a) 3 and 11 but not of 9.
 - b) 9 and 11. Are they multiples of 3?



4.- PRIME FACTOR DECOMPOSITION OF A NUMBER

Cada número compuesto puede escribirse como un producto de números, a veces incluso como varios productos distintos:

Example: $15 = 5 \times 3$

$$24 = 2 \times 12 = 3 \times 8 = 3 \times 2 \times 4 = 24 \times 1 = \dots$$

Pero cada número puede ser escrito únicamente como un producto de números primos que es único. Encontrar ese producto es lo que llamamos descomposición en factores primos. In English we call it **prime factor decomposition of a number**.

Si tenemos un número pequeño podemos hacer la descomposición mentalmente, pero recuerda que sólo puedes usar números primos:

Example: $6 = 2 \times 3$; $24 = 4 \times 6 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Solved Example:

Work out the prime decomposition of 3600:

The diagram illustrates the prime factor decomposition of 3600. On the left, a ladder method is shown where 3600 is divided by 2 repeatedly until it becomes 225, then 225 is divided by 3 repeatedly until it becomes 25, and finally 25 is divided by 5 repeatedly until it becomes 1. On the right, the same process is shown as a list of factors: 3600 is divided by 2 four times and by 5 twice. Below the diagrams, the final prime factor decomposition is given as $3600 = 2^4 \times 3^2 \times 5^2$.

Hint: If the number ends in zero, you can change each zero by the factors 2×5 , so if the last two digits are zeros, the prime decomposition will have $2^2 \times 5^2$.

Truco: Cuando el número acabe en 0, se puede cambiar cada cero por los factores 2×5 , así que si las dos últimas cifras son cero la descomposición en factores primos tendrá $2^2 \times 5^2$.

Solve the following exercises:

1. Work out the prime factor decomposition of the following numbers:

a) 108

b) 99

c) 42



d) 37

e) 100

f) 840

2. Complete these prime factor decompositions:

$$a) 360 = 2^? \times ? \times 5$$

$$b) 300 = ? \times ? \times 5^2$$

5. THE HIGHEST COMMON FACTOR AND THE LEAST COMMON MULTIPLE

5.1. Concept of the highest common factor (HCF)

To understand the concept, check this example to calculate HCF (30, 48, 54).

Firstly, calculate:

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Now, we choose the common factors: (Ahora elegimos los divisores comunes a los tres números)

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Which is the biggest one? It is 6, so:

$$\text{HCF}(30, 48, 54) = 6$$

Definition:

The highest common factor of several numbers is the largest number that evenly divides into all of them.

Spanish: _____



5.2. Rule for calculating the H.C.F.

A veces puede llevar mucho tiempo averiguar todos los divisores de varios números, por lo que hace falta un método más sencillo.

Rule:

"To work out the HCF of several numbers, first you have to find the prime factor decomposition of the given numbers and then, take the common factors with the lowest index".

Spanish: _____

Solved example:

Find out the HCF of numbers 36, 48 y 90.

1st step: Write them as a product of prime factors:

$$36 = 2^2 \cdot 3^2$$

$$48 = 2^4 \cdot 3$$

$$90 = 2 \cdot 3^2 \cdot 5$$

2nd step: Take the common factors with the lowest index:

$$\text{H.C.F.} = 2 \cdot 3 = 6$$

We can also do it in the English way. It consists in writing all the factors of each number in a row, and then mark the common ones.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$$

$$90 = 2 \cdot 3 \cdot 3 \cdot 5$$

We mark the factors that are common in all three numbers:

Spanish: Señalamos los factores que sean comunes en los tres números:

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$$

$$90 = 2 \cdot 3 \cdot 3 \cdot 5$$

$$\Rightarrow \text{H.C.F.} = 2 \cdot 3 = 6$$

Solve the following exercises:

1. Work out the factors of the numbers below and then find out the HCF:

a) 2 and 16

b) 3 and 25

c) 9, 12 and 18

d) 27, 36 and 63



2. Find out the HCF of the following numbers using the Spanish and the English methods:

a) 4, 6, 18 and 32

b) 3, 4, 12, 36 and 48

5.3. Concept of the least common multiple (lcm)

In this case, we calculate l.c.m. (2,3):

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,...

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36,....

Now, we choose the common multiples:

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,...

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36,....

Which is the smallest? It is 6, so:

$$\text{l.c.m. (2,3)} = 6$$



Definition: The least (lowest) common multiple of several numbers is the smallest number that is multiple of all of them.

And in Spanish: _____

5.4. Rule for calculating the l.c.m.

Rule:

"To work out the lcm of several numbers, first write them as a product of their prime factors and then take the common and non-common factors with the highest index."

And in Spanish: _____

Solved example:

Find out the lcm of numbers 36, 48 and 90:

1st step: First obtain the prime factor decomposition:

$$36 = 2^2 \cdot 3^2$$

$$48 = 2^4 \cdot 3$$

$$90 = 2 \cdot 3^2 \cdot 5$$

2nd step: Now, take the common and non-common factors with the highest index:

$$l.c.m = 2^4 \cdot 3^2 \cdot 5 = 720$$

A pesar de tener estas reglas es una buena idea acostumbrarse a calcular el m.c.d. y m.c.m. mentalmente cuando los números son pequeños. Sólo tienes que pensar en un múltiplo pequeño o en un divisor grande de los números dados.

Solved example:

Find out mentally the HCF and the lcm of the numbers below:

a) 3 and 5; HCF = 1 and lcm = 15

b) 2 and 4; HCF = 2 and lcm = 4

c) 6 and 15; HCF = 3 and lcm = 30



Solve the following exercises:

1. Work out the l.c.m. of the numbers below:

a) 9, 12 and 18

b) 27, 36 and 63

2. Work out the l.c.m. of the following numbers. What conclusion do you reach?

a) 2, 4, 8 and 16

b) 3, 4, 6 and 12

6. THE INTEGERS**6.1. The negative numbers**

Hay situaciones que no pueden ser expresadas con los números naturales, porque cuando expresamos una cierta cantidad necesitamos indicar la dirección en relación al origen. En estos casos se usan los números negativos, por ejemplo:

The number 0 is neither a positive nor a negative number.



6.2. The integer set

The integer set is formed by the natural numbers, zero and negative numbers. It is represented by $\mathbb{Z}$.

In Spanish: _____

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

The integer set doesn't have a beginning or an ending and it has infinite elements.

Solve the following exercises:

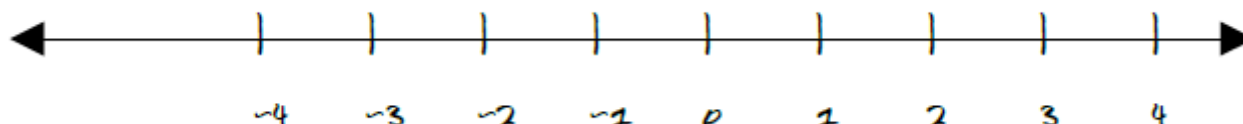
Write with an integer the following pieces of information:

- The plane is flying at 9500 m height.
- The minimum temperature yesterday was 3 degrees below zero.
- The garage is on the second cellar.
- The diver is swimming 20 metres deep.
- Sergio owes 25 euros.

6.3. Order of the integers.

6.3.1. Graphical representation.

Integers can be represented on a line, like this:



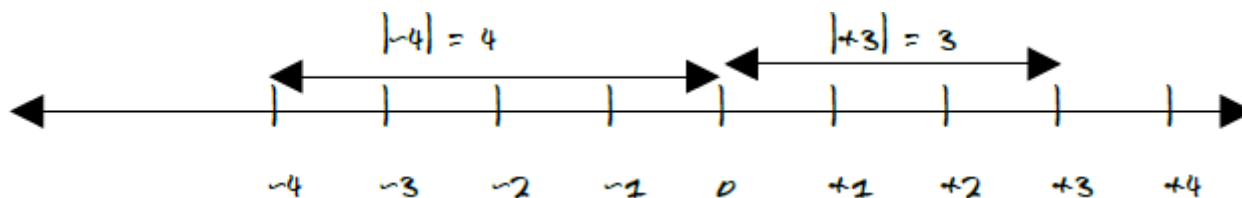
6.3.2. Absolute value and opposite.-

The **absolute value** of an integer is the number without the sign. It is represented as $|a|$ and it is called absolute value of a .

(In Spanish: El valor absoluto de un entero es el número pero sin signo. Se representa por $|a|$ y se lee valor absoluto de a)

$$|+5| = 5 \quad |-3| = 3 \quad |-7| = 7 \quad |+2| = 2$$

Geométricamente, el valor absoluto representa la distancia del número al cero:



The **opposite** of an integer is another integer with the same absolute value but different sign.

$$\text{Op } (+3) = -3 \quad \text{Op } (-5) = +5 \quad \text{Op } (-7) = +7 \quad \text{Op } (+2) = -2$$

Solve the following exercises:

1. Work out the absolute value of these numbers

a) $|-9|$ b) $|+5|$ c) $|-3|$ d) $|+7|$ e) $|0|$

2. Find out the number that has an absolute value of 7 and is between -8 and 3.

3. Work out the opposite of the following numbers:

a) $\text{Op } (-4) =$ b) $\text{Op } (+8) =$ c) $\text{Op } (15) =$ d) $\text{Op } (-301) =$

4. Write down the absolute value of the opposite of these numbers:

a) $|\text{Op } (+4)| =$ b) $|\text{Op } (-11)| =$ c) $|\text{Op } (-200)| =$ d) $|\text{Op } (+1001)| =$

5. Write down the opposite of the absolute value of these numbers:

a) $\text{Op } |+4| =$ b) $\text{Op } |-11| =$ c) $\text{Op } |-200| =$ d) $\text{Op } |+1001| =$

6.3.3. Order in the integer set

Solve the following exercises:

1. Write down the "less than" sign ($<$) or the "greater than" sign ($>$) where it corresponds:

a) $+4 \underline{\hspace{1cm}} +1$ b) $-1 \underline{\hspace{1cm}} -6$ c) $0 \underline{\hspace{1cm}} +3$ d) $-8 \underline{\hspace{1cm}} +2$ e) $-2 \underline{\hspace{1cm}} 0$

2. Sort out these negative numbers, greatest first: -5, -1, -2, -25

3. Sort out these numbers from the lowest to the highest: -4, +7, -6, -3, +5

6.4. ADDITION AND SUBTRACTION OF INTEGERS

Addition of Signed Numbers:

- To add two integers with the same signs, you must add their absolute value and leave the same sign:



$$\leftarrow 1 \rightarrow + \leftarrow 7 \rightarrow = +8$$

$$\leftarrow 12 \rightarrow + \leftarrow 8 \rightarrow = -20$$

- To add two integers with different signs, you must subtract their absolute values and leave the sign of the one with the greatest absolute value.

$$\leftarrow 15 \rightarrow + \leftarrow 7 \rightarrow = +8$$

$$\leftarrow 13 \rightarrow + \leftarrow 6 \rightarrow = -7$$

Subtraction of Whole Numbers:

- To subtract two integers, you must add the first one and the opposite of the second one.

$$3 - 4 = 3 + \leftarrow 4 \rightarrow = -1$$

$$\leftarrow 2 \rightarrow - \leftarrow 3 \rightarrow = \leftarrow 2 \rightarrow + \leftarrow 3 \rightarrow = -5$$

Solve the following exercises:

1. Work out the following sums with integers:

a) $-2 - 8 =$

b) $6 - 7 =$

c) $-19 + -20 =$

d) $-10 - (-4) =$

e) $3 - (-9) =$

f) $16 - (-2) =$

2. Work out the following combined additions:

a) $-2 - 8 + -3 - (-7) + (-2) - 4 =$

b) $6 - (+7) + (-3) - (-5) + 7 - (-8) =$

3. Work out the following combined additions:

a) $(-3) + (-7) - (+3) - (-3) + (-5) =$

b) $-5 + (-3) - (-2) + (-5) + (-3) =$

c) $-(-5) + (-2) + (-3) + (-7) - (-2) =$

d) $-(+4) - (-4) + (-3) + (-2) - (-7) =$

6.5. MULTIPLICATION AND DIVISION

6.5.1.- Multiplication of integers

To multiply two or more integers we have to multiply the signs and then the absolute value of the numbers.



To multiply the signs we use **the rule of signs**:

$$(+)\times (+) = (+)$$

$$(-)\times (+) = (-)$$

$$(+)\times (-) = (-)$$

$$(-)\times (-) = (+)$$

For example:

$$2 \times -5 = -10;$$

$$-3 \times -5 = 15;$$

$$-3 \times 7 \times -1 \times 2 = 42$$

6.5.2. The division of integers

To divide two or more integers we have to divide the signs and then the absolute value of the numbers. To divide the signs we use the rule of signs, again:

$$(+)\div (+) = (+)$$

$$(-)\div (+) = (-)$$

$$(+)\div (-) = (-)$$

$$(-)\div (-) = (+)$$

For example:

$$10 : -5 = -2;$$

$$-15 : -3 = 5$$

$$7 : -1 = -7$$

$$-10 : -2 = 5$$

Los enteros no cumplen la ley de composicion interna para la division ya que a veces la división de dos números enteros no es otro número entero. For example: $+10 : -3$

Solve the following exercise:

Work out the missing numbers:

a) $(-2) \times \underline{\hspace{1cm}} = -22$

b) $(-15) : \underline{\hspace{1cm}} = 15$

c) $\underline{\hspace{1cm}} \times (-5) = 30$

d) $\underline{\hspace{1cm}} : (+5) = -10$

e) $(+6) \times \underline{\hspace{1cm}} = -18$

f) $(-70) : \underline{\hspace{1cm}} = -7$

6.6. COMBINED OPERATIONS

6.6.1. Distributive property

Igual que los números naturales, los enteros satisfacen la propiedad distributiva esto es, la adición de dos números enteros multiplicado por otro numero entero es igual a la suma del número multiplicado por cada sumando del paréntesis.

For example: $-2 \times (3 - 5) = -2 \times 3 - 2 \times (-5) = -6 + 10 = +4$.

So, if you have to multiply an integer by an addition in the brackets, there are two ways of doing this:

First, the brackets and then the multiplication:

$$-5 \times (-3 + 5) = -5 \times 2 = -10$$



or applying the distributive property:

$$-5 \times (-3 + 5) = 15 - 25 = -10$$

Solve the following exercises:

1. Find out the result in two different ways:

a) $3 \times (-7 - 10) =$

b) $(-5) \times (12 - 4) =$

2. Find out the result using the distributive property:

a) $(-9) \times (8 - 9) =$

b) $2 \times (-10 + 3) =$

c) $4 \times (-5 + 9 - 6) =$

d) $(-9 + 7 - 2) \times (-8) =$

3. Complete the following exercises:

a) $(-5) \times (9 - 4) = -5 \times \underline{\quad} = -25$

b) $\underline{\quad} \times (5 - 7) = -10 + \underline{\quad} = \underline{\quad}$

c) $9 \times 8 = 9 \times (5 + \underline{\quad}) = \underline{\quad} + 27$

d) $3 \times (-6 + \underline{\quad}) = 3 \times \underline{\quad} + 3 \times (-9) = \underline{\quad}$

6.6.2. Extracting a common factor

En la suma $8 \times (-3) + 8 \times 5$ hay un factor que se repite en los dos sumandos que es el 8. Si aplicamos la propiedad distributiva pero al revés podríamos escribir:

$$8 \times (-3) + 8 \times 5 = 8 \times (-3 + 5)$$

$$36 - 24 + 12 = 4 \times 9 + 4 \times (-6) + 4 \times 3 = 4 \times (9 - 6 + 3)$$

Este paso de suma a producto se llama **sacar factor común**. In English it is called **Extracting the common factor**.

Solve the following exercises:

1. Extract a common factor of these operations and work out the result:

a) $(-5) \times 7 + (-5) \times (-12) =$

b) $(-2) \times 7 + (-2) \times (-3) =$

c) $5 \times 9 + 5 \times (-11) =$

d) $(-9) \times (-12) + (-9) \times 13 =$



e) $7 \times 2 + 7 \times (-21) =$

f) $(-2) \times 7 + (-2) \times (-3) =$

6.6.3. Order of the operations

When expressions have more than one operation, we have to follow rules for the order of operations. These are the same rules as for natural numbers:

Rule 1: First do any calculations inside the brackets.

Rule 2: Do the powers and roots.

Rule 3: Next do all the multiplications and divisions, working from left to right.

Rule 4: Finally, do all the additions.

Ahora en el paso final de la suma ya no importa el orden pero una buena idea es sumar todos los positivos, todos los negativos y después sumar al final el negativo y el positivo.

Solved Example:

Using the order of operations, work out $(-3) - 6 \cdot (-5 + 4) : (-3) + 7$:

Step 1: $(-3) - 6 \cdot (-5 + 4) : (-3) + 7 = (-3) - 6 \cdot (-1) : (-3) + 7$ Brackets

Step 2: $(-3) - 6 \cdot (-1) : (-3) + 7 = (-3) + 6 : (-3) + 7$ Multiplication

Step 3: $(-3) + 6 : (-3) + 7 = -3 - 2 + 7$ Division

Step 4: $-3 - 2 + 7 = -5 + 7 = 2$ Addition

Solve the following exercises

1. Find out the result:

a) $32 + (-12) : 6 =$

b) $(-8) \times 9 - 15 \times (-3) =$

c) $(-4) \times 10 : 2 + 14 : (-7) =$

d) $27 : (-3) \times 2 - (-4) =$

e) $(-18) : 6 + 5 \times (-10) =$

f) $18 : 9 + 5 - [(-15) \times 3 + 12 \times 4] =$

g) $(-6) \times [4 - (-2)] + [-8 + (-3) \times 2] =$

h) $(-35) : (5 + 2) + (-4) \times 9 - (7 - 2 \times 5) =$

i) $[(3 - 4) + (-2)] \times 4 + 9 : (-3) \times 6 =$

j) $\{ [(-12) - (-3)] \times 8 \} + 24 : \{ [(-2) + (-6)] : 2 \} =$



EXERCISES

UNIT 1. DIVISIBILITY AND INTEGERS.

Exercise 1. Write:

- a) The first five multiples of 11
- b) The multiples of 20 between 150 and 210
- c) A multiple of 13 between 190 and 200

Answer: a) 11, 22, 33, 44 y 55; b) 160, 180 y 200; c) 195;

Exercise 2. Write

- a) All the pairs of numbers whose product is 80.
- b) All the divisors of 80

Answer: a) $1 \cdot 80 = 2 \cdot 40 = 4 \cdot 20 = 5 \cdot 16 = 8 \cdot 10$; b) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80;

Exercise 3. Find all the divisors of

- a) 30 b) 39 c) 45 d) 50

Answer: a) 1, 2, 3, 5, 6, 10, 15, 30; b) 1, 3, 13, 39; c) 1, 3, 5, 9, 15, 45; d) 1, 2, 5, 10, 25, 50

Exercise 4. Find, in each case, all the possible values of "a" so that the result is at the same time a multiple of 2 and of 3:

- a) $4a$ b) $32a$ c) $24a$

Answer: a) 2 y 8; b) 4; c) 0 y 6;



Exercise 5. Find the prime numbers greater than 25 and less than 45.

Answer: 29, 31, 37, 41 y 43

Exercise 6. Classify these numbers into prime numbers and composite numbers:

14 17 28 29 47 53 57 63 71 79 91 99

Answer: prime numbers: 17, 29, 47, 53, 71 y 79; composite numbers: 14, 28, 57, 63, 91 y 99

Exercise 7. Select at a single glance,

$$a = 2^5 \cdot 3 \quad b = 2^2 \cdot 7^2 \quad c = 2 \cdot 3^2 \cdot 5 \quad d = 2^2 \cdot 5 \cdot 11 \quad e = 3 \cdot 5^2 \cdot 13 \quad f = 2^2 \cdot 3^2 \cdot 7$$

- | | |
|---------------------------------|---------------------------------|
| a) All the multiples of 10 | b) All the multiples of 14 |
| c) All the multiples of 15 | d) All the multiples of 18 |
| e) One that is a multiple of 13 | f) One that is a multiple of 30 |

Answer: a) c y d; b) b y f; c) c y e; d) c y f; e) e; f) c;

Exercise 8. Select at a glance,

$$a = 2 \cdot 3 \quad b = 2 \cdot 5 \quad c = 3 \cdot 5 \quad d = 2^2 \cdot 3 \quad e = 2^2 \cdot 5 \quad f = 2 \cdot 5^2$$

- a) The factors of 20 = $2^2 \cdot 5$
 b) The factors of 30 = $2 \cdot 3 \cdot 5$
 c) The factors of 60 = $2^2 \cdot 3 \cdot 5$
 d) The factors of 90 = $2 \cdot 3^2 \cdot 5$

Answer: a) b y e; b) a, b y c; c) a, b, c, d y e; d) a, b y c;

Exercise 9. Work out the lowest common multiple of a and b in each case:

- a) $a = 48$; $b = 56$ b) $a = 80$; $b = 88$ c) $a = 175$; $b = 350$

Answer: a) 336; b) 880; c) 350;

Exercise 10. Calculate the greatest common factor of a and b in each case:

- a) $a = 63$; $b = 84$ b) $a = 105$; $b = 120$ c) $a = 165$; $b = 198$



Answer: a) 21; b) 15; c) 33;

Exercise 11. Work out:

a) $\text{lcm}(2, 4, 8)$

b) $\text{GCF}(2, 4, 8)$

c) $\text{lcm}(10, 15, 20)$

d) $\text{GCF}(10, 15, 20)$

e) $\text{lcm}(20, 30, 40)$

f) $\text{GCF}(20, 30, 40)$

Answer: a) 8; b) 2; c) 60; d) 5; e) 120; f) 10;

Exercise 12. Find out the values of a and b knowing that their $\text{lcm}(a, b) = 20$ and their $\text{HCF}(a, b) = 2$

Answer: (10 and 4) or (20 and 2)

Exercise 13. Find all the possible forms of making piles of equal size with 72 sugar cubes.

Answer: 72 of 1; 36 of 2; 24 of 3; 18 of 4; 12 of 6; 9 of 8; 8 of 9; 6 of 12; 4 of 18; 3 of 24; 2 of 36; 1 of 72

Exercise 14. Ricardo can arrange his collection of picture cards into pairs, trios and groups of five. How many cards has Ricardo got if we know that he has more than 80 and less than 100?



Answer: Ricardo owns 90 picture cards

Exercise 15. A glass weighs 75 gr. and a cup weighs 60 gr. How many glasses do you have to put on a balance plate and how many cups on the other so that they are in balance?

Answer: 4 glasses and 5 cups;

Exercise 16. A market vender exchanges with a mate a batch of t-shirts for 24 € per piece for a batch of trainers for 30 € per piece. How many t-shirts does he give and how many pairs of trainers does he receive?

Answer: 5 t-shirt for 4 pairs of trainers

Exercise 17. In a timber yard there is a stack of pine planks. Each plank is 35 mm. thick and the stack is the same height as a stack of oak planks which are 90 mm. thick. What is the height of both stacks? (Give at least three solutions)

Answer: they must be multiples of 14 cm.

Exercise 18. A group of 60 kids accompanied by 36 parents go to a camp in the mountains. To sleep they agree to have the same number of people in each cabin. Also, the fewer cabins they occupy the less they pay. On the other hand the parents don't want to sleep with the kids, and the kids do not want to sleep with their parents. How many people will sleep in each cabin?



Answer: 12 people in each cabin

Exercise 19. Solve explaining the process

a) $16 + (-5) \cdot (+4) =$

b) $20 - (-6) \cdot (-4) =$

c) $(-2) \cdot (-5) + (+4) \cdot (-3) =$

d) $(-8) \cdot (+2) - (+5) \cdot (-4) =$

e) $10 + (-4) \cdot (+2) - (+6) =$

f) $(-5) - (+4) \cdot (-3) - (-8) =$

g) $14 - (+5) \cdot (-4) + (-6) \cdot (+3) + (-8) =$

h) $(+4) \cdot (-6) - (-15) - (+2) \cdot (-7) - (+12) =$

Answer: a) -4; b) -4; c) -2; d) +4; e) -4; f) +15; g) +8; h) -7;

Exercise 20. Work out

a) $8 + (4 - 9 + 7) \cdot 2 + 4 \cdot (3 - 8 + 4) =$

b) $4 \cdot [(+5) + (-7)] - (-3) \cdot [7 - (+3)] =$

c) $(-3) \cdot (+11) - [(-6) + (-8) - (-2)] \cdot (+2) =$

d) $(-6) \cdot [(-7) + (+3) - (7 + 6 - 14)] - (+7) \cdot (+3) =$

Exercise 21. Alexander the Great, one of the greatest generals in history, was born in 356 B. C. and died in 323 B. C. At what age did he die? How many years have passed since he died?

Answer: He died when he was 33 years old, 2,331 years ago (the year 0 doesn't exist; Jesus was born in the year 1).



Exercise 22. A water park businessman does this summary of his finances throughout the year:

January - May → Monthly losses of 2,475 €

June - August → Monthly earnings of 8,230 €

September → Earnings of 1,800 €

October - December → Monthly losses of 3,170 €

What was the balance at the end of the year?

Answer: 12,835 €

Maths is the mother of all sciences

