UNIT 9

ALGEBRA AND EQUATIONS.

1. VARIABLES.

A variable is a symbol that represents a number.
We usually use letters such as x, n, p, t for variables.

Letters are useful if we want to operate with an unknown number instead of using a specific one. When letters express numbers they have the same operating properties.

Let's look at some examples:

When we say that *s* represents the side of a square, then *s* represents a number, and

4s is the perimeter of the square, and

$s \cdot s$ is the area of the square.

The part of mathematics that deals with the study of expressions with letters and numbers is called **algebra**.

2. EXPRESSIONS.

An expression is a mathematical statement with numbers and variables.

Examples:

3 x x + 3 $2 \cdot (x - 5)$ $x^2 - 3x$

If Mark weighs 80 kg and he gains n kg, his weight is now 80 + n.

Exercise 1. Calling "a" the age of a woman, write an expression for:

a) The age she will be in 2015.

b) The age she was 7 years ago

c) The age she will be after living the same time again.

Exercise 2. Calling "x" a number, express in algebra:

a) The sum of the number and 10

b) The difference between 123 and the number

c) Double the number

d) Triple the number plus three units

e) Half the number minus seven

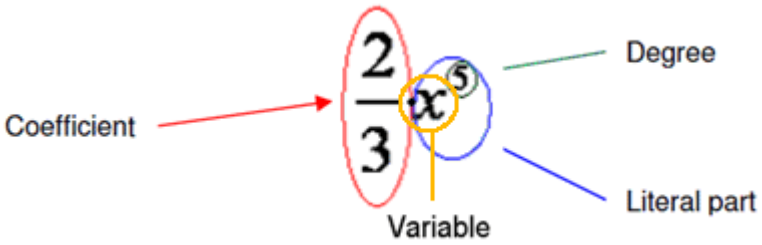
f) Three quarters of the number plus forty-six

3. MONOMIALS.

The simplest algebraic expression formed by products of numbers and letters is called a **monomial**.

A monomial consists of the product of a known number (the **coefficient**) by one or several letters (the **literal part**) with exponents that must be *constant and positive whole numbers*.

Generally, the product signs are not included in monomials, so when we find a number followed by one or more letters, we understand that they are multiplied.



Examples:

a) $2x$ is a monomial. 2 is its coefficient, x is its literal part.

b) $-3x^2$ is a monomial, - 3 is its coefficient, x^2 is its literal part; x is the variable, and the degree is 2.

c) $\frac{3}{5}t^7$ is a monomial, $\frac{3}{5}$ is its coefficient, t^7 is its literal part; t is the variable and the degree is 7.

d) $5xy^2$ is a monomial, 5 is its coefficient, xy^2 is its literal part, x and y are the variables and the degree is 2.

e) $2x+7$ is an algebraic expression but it is not a monomial, because it is made up of two monomials.

f) $\frac{3}{x}$ is an algebraic expression but it is not a monomial, because the exponent of the literal part is not a positive number ($\frac{3}{x} = 3x^{-1}$).

Exercise 3. Find which of the following expressions are monomials; if they are, determine their coefficient, literal part and variables:

a) $-\frac{1}{5}x^7$

b) $2t^2$

c) $a+b$

d) a^9

e) $\sqrt{2}n^4$

f) $3\sqrt{n}$

g) $7abc^2$

4. OPERATIONS WITH MONOMIALS.

4.1. Addition or subtraction of two monomials.

We add or subtract the coefficients and we leave the literal part unchanged.

Two monomials can only be simplified when both of them have the same literal part, that is, when they are **like monomials**. When the literal part is different, the addition is left indicated.

Examples:

a) $5x+7x=12x$

b) $\frac{1}{2}x+2x=\left(\frac{1}{2}+2\right)x=\frac{5}{2}x$

c) $7ab+2ab=9ab$

Exercise 4. Calculate:

a) $2x-7x=$
b) $15x+2x=$
c) $8x-x=$
d) $\frac{15}{2}x+2x=$
e) $-2x^2+23x^2=$
f) $15x+2=$
g) $3a-7a=$
h) $x^2+3x=$
i) $x+\frac{2}{3}x=$

4.2. Product of two monomials.

We multiply the coefficients and also the literal part (remember how we multiply powers with the same base).

Examples:

a) $5x\cdot 7x=35x^2$
b) $\frac{1}{2}x^2\cdot 2x=x^3$
c) $7a^3\cdot 2a^2=14a^5$

Exercise 5. Calculate:

a) $2x\cdot 7x=$
b) $15x\cdot 2x=$
c) $8x\cdot x=$
d) $\frac{15}{2}x\cdot 2x^3=$
e) $-2x^2\cdot 23x^2=$
f) $15x\cdot 2=$

5. MANIPULATING ALGEBRAIC EXPRESSIONS.

5.1. Evaluating an expression.

Evaluating an expression at some number means replacing a variable in the expression with the number, and then if necessary calculating the final value.

Example:

Evaluate $5x+3$ when $x=2$

$5x+3$ becomes $5\cdot 2+3=13$

Remember the **BODMAS** rule: brackets, orders (powers and roots), divisions and multiplications, and finally, additions and subtractions.

Exercise 6. Evaluate these expressions:

a) $-\frac{1}{5}x^2$ when $x=3$

b) $2t^2$ when $t=2$

c) $a+b$ when $a=b=7$

d) a^9 when $a=-1$

e) n^4+3n when $n=3$

f) $\frac{3n+2}{n-3}$ when $n=5$

g) $7a+b-c^2$ when $a=3, b=-2, c=4$

5.2. Expansion of brackets.

> Multiplying a number by an addition is equal to multiplying by each summand and then adding the partial products.

Examples:

a) $2(x+6)=2x+12$
b) $3(2x-7)=6x-21$
c) $3x(3x+2)=9x^2+6x$

Exercise 7. Fill in the missing terms:

a) $5(3x-4) = \square - 20$

b) $x\,(x-2) = \square - 2x$

c) $7x\,(3x-y) = \square - 7xy$

d) $a\,(4x-a) = \square - a^2$

Exercise 8. Expand:

a) $2(x-4)=$

b) $6(2-x)=$

c) $6x(2x-1)=$

d) $2t(2t+8)=$

e) $5x(2x-3y)=$

f) $x(x-1)=$

g) $2(7+7x)=$

h) $2x(3x-a)=$

Exercise 9. Write down expressions for the area of these rectangles, and then expand the brackets as in the example:

a) 3x+1

3 $A = 3(3x+1) = 9x+3$

b) 2x-1

7 A =

c) 3x

x-2 A =

d) x-7

4 A =

5.3. Common factors.

It is important to know how to write expressions including brackets when it is possible; this is called **factorization** or **factoring**.

Examples:

$12x+6 = 6\cdot 2x + 6\cdot 1 = 6(2x+1)$

$= 6(2x+1)$

$5x^2 - 8x = x(5x-8)$

We can check our answer by expanding the expression.

$21x - 14 = 7\cdot 3x - 7\cdot 2 = 7(3x-2)$

$= 7(3x-2)$

Exercise 10. Factorise:

a) $10x+15=$

b) $4x-12=$

c) $9x-9=$

d) $3x^2-2x=$

e) $9x^2+3x=$

f) $3x-3=$

g) $33x^2-3x=$

h) $5x-3x^2=$

i) $13x^2-2=$

6. EQUATIONS.

An equation is a statement in which two expressions are equal.

The letter in an equation is called the **unknown** (sometimes it is called the **variable**).

Examples of equations:

$x=2$ $3=x$ $2x=4$ $5x+1=3$ $x^2=4$

In an equation we consider two members:

The first member is the member on the left-hand side.
The second member is the member on the right.

Each monomial is called a term. Monomials with the same literal part are called **like terms**.

Example:

In the equation $3t-2=-7$ we say:

- The first member is $3t-2$
- The second member is -7
- There are three terms $3t$, -2 and -7.
- The unknown is t.

In the equation $3x^2-6=5x+2$:

- The first member is $3x^2-6$
- The second member is $5x+2$
- There are four terms $3x^2$, -6, $5x$, and 2.
- The unknown is x.

7. SOLVING EQUATIONS.

Solving an equation is finding out a solution to it.
A solution to an equation is the number that makes the equality true when we replace the unknown or variable with that number.

Examples:

- 2 is a solution to the equation $-2x = 4$ because if we replace x by 2 that gives us $2 \cdot 2 = 4$, which is true.
- 3 is a solution to the equation $5x + 1 = 16$ because if we replace x by 3, we have $5 \cdot 3 + 1 = 16$, which is true.
- 4 is not a solution to $3x - 1 = 4$ because $3 \cdot 4 - 1 = 4$ is not true.

7.1. Rules for simplifying and solving equations.

These are the basic rules for simplifying equations:

These rules must be followed, usually in this order, to solve linear equations.

Rule 1 If there are any brackets we must remove them (**expand the expression**). If there are any like terms, we combine them doing all the common operations with numbers.

Rule 2 We move all the x-terms to one side and all the numbers to the other. To do this, we can add any number to both members of the equation, or subtract it, so that **any x-term or number that is adding** (positive sign) **moves to the other side subtracting** (negative sign), and vice versa.

Rule 3 We combine all the like terms doing all the common operations with numbers (again!).

Rule 4 Any number that is **multiplying the whole expression** on one side moves to the other side **dividing**, and a number that is **dividing the whole expression** on one side moves to the other **multiplying**.

Examples:

1. Solve: $5x + 12 - 4 = -2$
   Rule 1: we combine like terms adding the two numbers in the first member; we get: $5x + 8 = -2$
   Rule 2: now we may subtract 8 from each member and make the addition of the integer numbers that appear in each member: $5x + 8 - 8 = -2 - 8$
   … but it is easier to move the 8 to the right member as -8: $5x = -2 - 8$
   Rule 3: we combine like terms: $5x = -10$

Rule 4: we divide each member by 5, or we can say that the 5 which is multiplying moves to the second member dividing: $x = \frac{-10}{5}$ → x = –2

We can check the solution (always in the original equation):

5·(–2) + 12 – 4 = –2 → –10 + 12 – 4 = –2 → –2 = –2, so our solution is correct.

2. Solve 7x + 5 = 2x
3. Rules 2 and 3: we subtract 5 from each member and make the addition of the integer numbers that appear: 7x + 5 –5 = 2x –5 → 7x = 2x –5
   Then we subtract 2x from each member…: 7x –2x = 2x–5 –2x
   …and we add like terms: 5x = –5
   Rule 4: we divide each member by 5, and we get x = –1
   We check the solution in the original equation:
   7(–1) + 5 = 2(–1) → –7 + 5 = –2 → –2 = –2,
   so our solution is correct.

4. Solve: 2(x + 5) + 1 = 17
   Rule 1: we expand 2x + 10 + 1 = 17 and we combine like terms: 2x + 11 = 17
   Rule 2: we subtract 11 from each member: 2x + 11 – 11 = 17 – 11 or we can simply say that 11 moves to the right member as –11: 2x = 17 – 11.
   Rule 3: we combine like terms: 2x = 6
   Rule 4: we divide each member by 2 (or we can say that the 2 which is multiplying moves to the right member dividing): x = 3
   We check the solution: 2(3 + 5) + 1 = 17 → 2(8) + 1 = 17 → 16 + 1 = 17, so our solution is correct.

Exercise 11. Solve:

1. $x + 7 = 10$
2. $x - 3 = 21$
3. $t - 6 = 7$
4. $x - 22 = 13 - 4$
5. $4 + y = 54$
6. $12 = 3 + z$
7. $m - 24 = 17$
8. $1 + p = 2 - 89$
9. $x + x = 37 - 2$

10. $23 + y + 2y = 3$

11. $4x = 29 - 1$

12. $100 = 16x + 20$

13. $5x + 3 = 20$

14. $10x = 13 - 2$

15. $2x + 7 = 6$

16. $2 = 7x - 3$

17. $12x - 7 = 13$

18. $3x - 5x = 23$

19. $5x + 7x = 23 - 1$

20. $44 + x = 12 - 3x$

Exercise 12. Solve:

1. $3(x-2) = 7$

2. $2(x+3) = 2$

3. $6(x-5) = -2$

4. $7(x-1) + 2x = 20$

5. $3x - 2(2x+3) = 0$

6. $4(3x-5) - x = 100$

7. $-21 = 6(3x-1) - 13x$

8. $9(1-2x)+22x=1$

9. $2(3-x)+5x=1-3x$

10. $(3-x)\cdot 2-15x+2=3(1-3)+2x-1$

Exercise 13. Express each problem as an equation and solve it:

1. The sum of my age and 7 is 42. Find my age.

2. The difference of a number and 23 is 124. Find the number.

3. The quotient of 35 and a number is 7. Find the number.

4. Is someone gives me €24 I will have €34.23. Find how much money I have.

5. Double my age minus 7 years is the age of my elder brother, who is 19 years old. How old am I?

6. The area of a square is 144 square metres. Find its length.

7. A teacher gave a number of coloured pencils to each of 8 girls, except to one of them, who only received 4 pencils. The teacher gave 53 pencils in total. How many pencils did each girl receive?

8. Donald thinks of a number, multiplies it by 3 and subtracts 7. His answer is double the number plus 5 units. Which is the number?

9. In a triangle, the smallest angle is 20° less than the middle angle, and the largest angle is twice the middle one. Find all the angles.

10. If we shorten two opposite sides of a square by 2 cm, we get a rectangle with an area which is 28 cm^2 less than the area of the square. Find out the perimeter of the rectangle.

EQUATIONS

15. (page 165). Solve the following equations:

a) $2x+5(3x-1)=x-13$

b) $5-4(2x-3)=2x+7$

16. (page 165). Solve these equations:

a) $7x-5(3x+2)=x-4$

b) $7x+9-5x=3(2x-1)+2$

17. (page 165). Solve the equations below:

a) $5x-3(4x-2)=4(2x-1)$

b) $5-4(3x+2)=4-5(3x-1)$

18. (page 165). Solve these equations:

a) $4(3x+1)-4x=8-2(x-3)$

b) $5x-3(2x-1)-(x+5)=1-2(3x+5)$

19. (page 165). Solve these equations:

a) $\frac{x}{2}+\frac{1}{4}=\frac{13}{4}$

b) $\frac{5}{6}-\frac{4x}{3}=\frac{1}{6}$

20. (page 165). Solve these equations:

a) $\frac{x}{6}+\frac{4x}{3}=\frac{5}{2}$

b) $\frac{5x}{4}-\frac{x}{8}=\frac{9}{4}$

21. (page 165). Solve these equations:

a) $\frac{3x}{2}-\frac{11}{4}=\frac{x}{4}-2$

b) $\frac{4x}{3}+5=\frac{x}{3}+\frac{13}{3}$

22. (page 165). Solve these equations:

a) $\frac{5x}{2}-\frac{2x+3}{6}=\frac{5}{3}$

b) $\frac{2x}{3}-\frac{5x-7}{6}=\frac{x}{2}+\frac{5}{3}$

23. (page 165). Solve these equations:

a) $\frac{4x}{3}-5=\frac{26}{9}-\frac{3x-4}{9}$

b) $\frac{2x-1}{4}+2-\frac{3x}{8}=x+\frac{7}{3}$

24. (page 165). Solve these equations:

a) $x-\frac{2x-3}{5}-4=\frac{5x+1}{6}-\frac{47}{12}$

b) $\frac{3x-1}{6}-2x=\frac{19}{24}-\frac{4x+5}{8}$

45. (page 169). Solve the following equations:

a) $3x+2(4x-1)=x+18$

b) $1-3(x+1)=2x+13$

46. (page 169). Solve the following equations:

a) $5x-4(2x+3)=2x-17$

b) $4x+5-7x=2(3x-6)-1$

47. (page 169). Solve the following equations:

a) $7x-4(2x-5)=3(5x-2)-6$

b) $4-5(2x+1)=-3(4x-5)$

48. (page 169). Solve the following equations:

a) $9x-5(2x-1)=-3(x+4)$

b) $7x+3(5x-3)-(5x+1)=7(2x+2)$

49. (page 169). Solve the following equations:

a) $\frac{x}{3}+\frac{1}{2}=\frac{3}{2}$

b) $\frac{2}{5}-\frac{3x}{4}=\frac{17}{5}$

50. (page 169). Solve the following equations:

a) $\frac{7x}{2}-\frac{5x}{3}=\frac{11}{6}$

b) $\frac{5x}{4}-\frac{x}{2}=-\frac{3}{2}$

51. (page 169). Solve the following equations:

a) $\frac{5x}{2}+\frac{7}{4}=5+\frac{x}{6}+\frac{1}{4}$

b) $\frac{2x}{5}-\frac{7x}{2}-4=\frac{x}{2}+\frac{7}{5}$

52. (page 169). Solve the following equations:

$\frac{3x}{2}+\frac{4x+1}{3}=-\frac{5}{2}$

a) $\frac{4x}{3}-\frac{2x-5}{2}=\frac{3x}{4}$

53. (page 169). Solve the following equations:

a) $\frac{x}{6}-\frac{2x-3}{2}+\frac{3}{4}=\frac{5x-2}{3}$

b) $\frac{2x}{3}-\frac{4x+5}{6}=\frac{7x-1}{3}+\frac{1}{2}$

54. (page 169). Solve the following equations:

a) $2x-\frac{4x-3}{2}-5=\frac{6x+1}{3}-\frac{1}{6}$

b) $\frac{5x-3}{4}-3x=\frac{1}{2}-\frac{4x+5}{8}$

70. (page 170). Solve the equations below:

a) $5x-4(3x-1)-(6x+1)=5(3x+12)-1$

b) $7(3x-1)-5(4x+3)=2(3x+5)-5(3x+12)$

71. (page 170). Solve the equations below:

a) $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=26$

b) $-\frac{x}{2}+\frac{x}{3}-\frac{x}{4}+\frac{x}{6}=2$

72. (page 170). Solve the equations below:

a) $\frac{2x-3}{5}+2x-\frac{4x+7}{3}=-4$

b) $\frac{5x-4}{6}+2=2x-\frac{7x+1}{8}$

73. (page 170). Solve the equations below:

a) $\frac{x-1}{2}-\frac{x-2}{3}+\frac{x-3}{4}-\frac{x-4}{6}=\frac{2}{3}$

b) $\frac{3x-1}{2}-\frac{4x+2}{3}+\frac{5x-3}{4}-\frac{7x+4}{6}=-\frac{11}{4}$

PROBLEMS

25. (page 167). Solve in your mind using the trial-and-error method the following problems:

a) Oscar has 2€ more than his sister Sonia. If they have €16 between them, how much money has each one got?

b) If Dawn has €3 more than her cousin Charles and between both of them they have €13, how much money has each one got?

c) Martha has twice as much money (that is, double) as her brother Louis, and between them they have €15; how much money has each one got?

d) Julia has three times as much money as her cousin Mary. If they have €16 between them, how much money has each one got?

26. (page 167). Calculate two consecutive integer numbers whose sum is 57.

27. (page 167). Calculate a number when we know that that number plus half of it is 39.

28. (page 167). Susan has twice as much money as her cousin Thomas. If they have €70.2 between them, how much money has each one got?

29. (page 167). In an isosceles triangle each of the equal sides is 6 m more than the other side. If the perimeter is 36 m, how long is each side?

30. (page 167). Calculate the dimensions of a football pitch, knowing that its length is double its width, and its perimeter is 294 m.

55. (page 169). Solve in your mind using trial and error the following problems:

a) John has two more books than his cousin Susan. If they have 12 books between them, how many books has each one got?

b) If Ann has €3 more than her friend Louis and they have €11 between them, how much has each one got?

c) If Sonia has twice as much money as her brother Tony and they have €9 between them, how much has each one got?

d) Between Emma and her cousin Martha they have €20. If Emma has three times as much money as Martha, how much has each one got?

56. (page 169). Calculate two consecutive integer numbers whose sum is 61.

57. (page 169). Calculate a number knowing that that number plus half of it plus a third of it is equal to 22.

58. (page 169). John has €12 more than his cousin Hannah. If they have €63 between them, how much has each one got?

59. (page 169). Sarah has twice as much money as her cousin Alf. If they have €24.60, how much has each one got?

60. (page 170). Sylvia spends in the afternoon half her pocket money on a cinema ticket, and one sixth on sweets. If she still has €4, how much pocket money did she have in the morning?

61. (page 170) In a garden there are 91 trees, which include palm trees, willows and pine trees. If the number of palm trees is double that of willows and the number of pine trees is double that of palm trees, how many trees are there of each kind?

62. (page 170) Calculate three consecutive integer numbers whose sum is 45.

63. (page 170) Each side of a triangle is 5 m more than the previous one. If the perimeter is 37.5 m, how long is each of its sides?

64. (page 170) The perimeter of a rectangle is 26 m. The longest side is 3 m shorter than the smallest one; how long is each side?

79. (page 171) I bought a shirt and a jacket for €72. The jacket cost €12 more than the shirt. How much was each piece of clothing?

80. (page 171) How can we share €800 between Mary and John, so that Mary received €200 more than John?

81. (page 171) Find three consecutive integer numbers that add up to 72.

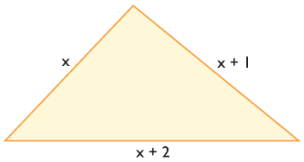
82. (page 171) A number plus double that number, plus half the same number add up to 112. Calculate the number.

83. (page 172) The longest sides of a romboid are 7.5 m longer than the shortest sides. If the perimeter is 115 m, how long is each side?

84. (page 171) An integer number plus double the next number equals 71. Calculate the number.

85. (page 171) In a school there are 17 more girls than boys, and there are a total of 1,087 students. How many are girls and how many are boys?

86. (page 171) The perimeter of this triangle is 27 m. How long is each side?



87. (page 171) A bus is carrying 10 times more people than a car. If they are carrying a total of 55 people, how many people is each carrying?

88. (page 171) A rectangular field measures 15 metres more on its longest side. If the perimeter is 170 m, calculate its length and width.

89. (page 171) Anthony, James and Sharon are security guards who have received €1,057 for a job. James has worked half as many days as Anthony and Sharon. How much has each one received?

90. (page 171) We have 113 oranges in 3 different boxes. The biggest box has 7 more oranges than the middle-sized box, and the middle-size one has 5 more than the smallest one. How many oranges has each box got?

91. (page 171) In a farm there are some rabbits and chickens. There are 55 heads and 160 legs. How many rabbits and chickens are there?

92. (page 171) Dawn has 13 more stickers than her sister Mary. If they have a total of 67 stickers, how many stickers has each one got?

93. (page 171) Calculate three consecutive even numbers which add up to 42.

94. (page 171) The three angles of a triangle are consecutive integer numbers. What are the three angles?

95. (page 171) I bought a pair of trousers, a pair of shoes and a tie for €72. The shoes cost twice as much as the tie, and the trousers the same as the shoes and the tie put together. How much did each item cost?

96. (page 172) How can we share €574 between Oscar, Sonia and Dawn, so that Sonia gets double what Oscar gets, and Alba gets double what Sonia gets?

97. (page 172) Find four consecutivos integer numbers which add up to 154.

98. (page 172) A number plus three times that number minus a third of the same number equals 33. Calculate that number.

99. (page 172) How can we share 28 chocolats between Martha, John and Louis, so that Juan gets half as much as Martha and Louis gets half as much as John?

100. (page 172) A rectangular field is twice as long as it is wide. If the perimeter is 270 m, calculate its length and width.

101. (page 172) In a parking lot there are cars and motorcycles, a total of 65 vehicles and 190 wheels. How many cars and how many motorbikes are there?

102. (page 172) Jane has €5 less than Ann, and Ann has €5 less than Anthony. If they have a total of €30, how much has each one got?

103. (page 172) Calculate three consecutive odd numbers which add up to 57.

104. (page 172) Paul read a quarter of the pages in a book in one day. The next day he read a third. If he still has 75 pages to read, how many pages has the book got?

105. (page 172) Albert climbs a mountain in 4 days. The first day he climbs a third of the way, the second day another third, the third day half of the rest of the way, and the fourth day 300 m. How high is the mountain?

106. (page 172) The sum of the perimeters of a square and an equilateral triangle is 56 cm. If the triangle and the square have sides the same size, how long is each side?

107. (page 172) Robert is three times as old as his son Julian; David, his youngest son, is half as old as Julian, and the sum of their ages is 63 years. How old is each one?

108. (page 172) With the money I have plus half what I have, plus half half what I have, plus 1 euro, I would have €64. How much money do I have?

109. (page 172) Christine bought some tuberose bulbs (=bulbos de nardos). When they grew, they split into two and she got twice as many bulbs. The next autumn she planted them again, and again all the bulbs split into two. How many bulbs did she buy, if that autumn her garden had 100 tuberoses?