



UNIT 8. PROPORTIONS.

1. RATIO AND PROPORTION. (= Razón y Proporción).

A **ratio** is a division of two comparable magnitudes. The ratio between two numbers is how many times one is bigger than the other.

Example: In Spanish: "La razón entre la masa de un saco grande y la de uno pequeño es $\frac{7.5}{2.5} = 3$, lo que significa que el saco grande tiene 3 veces la masa del saco pequeño"

A **ratio** is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:) or as a fraction.

Suppose we want to write the ratio of 8 and 12.

We can write this as $8 : 12$ or as $\frac{8}{12}$, and we say the ratio is *eight to twelve*.

Examples:

Janet has a bag with 4 pens, 3 sweets, 7 books, and 2 sandwiches.

1. What is the ratio of books to pens?

Expressed as a fraction, the answer would be $\frac{7}{4}$

Two other ways of writing the ratio are 7 to 4, and $7 : 4$.

2. What is the ratio of sweets to the total number of items in the bag?

There are 3 candies, and $4 + 3 + 7 + 2 = 16$ items total.

The answer can be expressed as $\frac{3}{16}$, 3 to 16, or $3 : 16$

A **proportion** is a name we give to a statement that two ratios are equal. It can be written in two ways:

- two equal fractions: $\frac{a}{b} = \frac{c}{d}$

or,

- using a colon: $a:b = c:d$



When two ratios are equal, then the cross products of the ratios are equal. That is, for the proportion, $a:b = c:d$, $a \times d = b \times c$

$$\frac{3}{5} = \frac{21}{35}$$

The following proportion is read as "twenty is to twenty-five as four is to five."

$$\frac{20}{25} = \frac{4}{5}$$

	<p>Example: $\frac{13.5}{5} = \frac{5.4}{2} = 2.7 \text{ €}$</p> <p>The five-litre bottle of oil costs € 13.6, and the two-litre bottle costs € 5.4. In both bottles you get the same price per litre, which is the proportion constant. (In Spanish: en las dos garrafas se obtiene el mismo precio por litro, que es la constante de proporcionalidad.)</p>
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In problems involving proportions, we can use cross products to test whether two ratios are equal and form a proportion. To find the cross products of a proportion, we multiply the outer terms, called the *extremes*, and the middle terms, called the *means*.

EXAMPLE: Find whether each of the following statements is a proportion:

a) $\frac{2}{3} = \frac{6}{9}$

Using cross products to verify: $2 \cdot 9 = 3 \cdot 6$.

So, it's a proportion.

b) $\frac{4}{3} = \frac{20}{18}$

Using cross products to verify: $4 \cdot 18 \neq 3 \cdot 20$.

So, it's not a proportion.

EXAMPLE: What value of "n" will make this a proportion?

$$\frac{6}{15} = \frac{n}{25}$$



EXERCISE 1. Find the unknown side in each ratio or proportion:

a) $\frac{1}{5} = \frac{3}{x}$

f) $\frac{5}{8} = \frac{8}{x}$

b) $\frac{1}{5} = \frac{5}{x}$

g) $\frac{1}{4} = \frac{3}{x}$

c) $\frac{1}{9} = \frac{5}{x}$

h) $\frac{2}{3} = \frac{5}{x}$

d) $\frac{2}{7} = \frac{x}{21}$

i) $\frac{12}{21} = \frac{16}{x}$

e) $\frac{3}{4} = \frac{7}{x}$

EXERCISES: Problem solving

1. Every week I eat 7 cakes

So every 2 weeks I eat _____ cakes
and every 3 weeks I eat _____ cakes.

2. For every 2 bags of crisps you buy you get 1 sticker,

For every 6 bags of crisps you buy you get _____ stickers.
To get 4 stickers you must buy _____ of crisps.

3. Colour 1 in every 3 squares black in this pattern.

Are there 2 black squares for every 6 squares? - yes / no.



4. Make a tile pattern where 1 in every 5 tiles is black.

5. Sarah uses 3 tomatoes for every
- $\frac{1}{2}$
- litre of sauce.

How many tomatoes does she need to for 1 litre of sauce?

How many tomatoes does she need to for 2 litres of sauce?

6. A mother seal is fed 5 fish for every 2 fish for its baby.

Alice fed the mother seal 15 fish.

How many fish did its baby get?



7. A mother seal is fed 5 fish for every 2 fish for its baby.
Alice fed the baby seal 8 fish.
How many fish did its mother get?
8. The juice of 5 oranges is used to make one jug of orangeade.
- a. How many oranges will you need to make 2 jugs of orangeade?
What operation do you need to use?
- b. How many jugs could you make using 36 oranges?
What operation do you need to use?

EXERCISES

1. Simplify the following ratios:

3 : 6

25 : 50

40 : 100

9 : 21

11 : 121

For some purposes the best is to reduce the numbers to the form $1 : n$ or $n : 1$ by dividing both numbers by either the left hand side or the right hand side number. It is useful to be able to find both forms, as any of them can be used as the unit in a problem.

Examples:

Which will be the divisor if we were to reach the form $1 : n$ for the ratio $4 : 5$?

The divisor will be 4 and the ratio will be $1 : 1.25$

And if we are to reach the form $n : 1$ for the same ratio?

The divisor will be 5, and the ratio will be $0.8 : 1$

2. Reduce to the form $1 : n$ and $n : 1$ 10 shovels of cement, 25 shovels of sand

3. Reduce to the form 1 : n the following:

$$4 : 9$$

$$7 : 5$$

$$30 : 100$$

$$5 : 12$$

4. Reduce to the form n : 1 the following:

$$14 : 9$$

$$15 : 5$$

$$40 : 100$$

$$75 : 12$$

2. DIRECT PROPORTION.

Two variables are in direct proportion to one another if they are always in the same ratio.

Values that are in direct proportion are often displayed in a table.

x	2	4	10
y	8	16	40

Notice that if x is doubled, then y is doubled too. All the values fit the equation $y = 4x$.

In other words:

This is the formal definition of direct proportion:

Two quantities, A and B, are in direct proportion if by whatever factor A changes, B changes by the same factor.

Here is a shorthand way to say that the quantities A and B are directly proportional:

$$A \propto B$$

The Greek letter between the A and the B is called alpha. It is here written in lower case script. In this context it is shorthand for the phrase "is **directly proportional to**". So, the above statement reads "A is **directly proportional to B**".



How to determine if two variables are directly proportional?

Think about these tables:

Y	1	2	3	4	5	6
X	2	4	6	8	10	12
$\frac{y}{x}$	0.5	0.5	0.5	0.5	0.5	0.5

Y	1	2	3	4	5	6
X	3	5	7	9	11	13
$\frac{y}{x}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{4}{9}$	$\frac{5}{11}$	$\frac{6}{13}$

FOR MORE EXERCISE PRACTICE ABOUT DIRECT PROPORTIONS, PLEASE VISIT THIS WEBPAGE.

http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/proporc_p.html



EXERCISE 5. Solve the following proportion (<http://www.321know.com/rat-prop-crossx.htm#section2>) by cross-multiplying. (Copy two examples into your notebook)

EXERCISE 6. Try the following interactive activity to understand ratios:

(<http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/flash1.shtml>)

(PERCENT AND FRACTIONS) AND (RATIO AND PROPORTION)

Related or independent amounts.



In the picture there is a group of 8 people queueing to buy a cinema ticket. If more people come, the queue will get longer.

If some people leave, the queue will get smaller. The number of people and the length of the queue depend on each other, that is to say, they are related.

When one thing depends on another thing, we say those things are related.

However, the number of people in the queue and the number of cars in the street are not related, and they do not depend on each other. We say that those amounts are independent.

EXERCISE 7.

Think about these things, and say if they are related or independent:

1. The number of hours I work and the money receive are...	<i>Related</i>	<i>Independent</i>
2. The number of wheels in a truck and its speed are...	<i>Related</i>	<i>Independent</i>
3. The weight and the volume of water are...	<i>Related</i>	<i>Independent</i>
4. A man's height and his intelligence are...	<i>Related</i>	<i>Independent</i>
5. The colour of a car and its maximum speed are...	<i>Related</i>	<i>Independent</i>
6. The speed of a train and the time it takes to go from a city to another are...	<i>Related</i>	<i>Independent</i>

Related and proportional amounts

TABLA 1

Número de huevos	1	2	3	4	5	6	7
Valor en pesetas	5	10	15	20	25	30	35

TABLA 2

1	2	3	4	5	6	7	× 5
5	10	15	20	25	30	35	

TABLA 3

: 5	1	2	3	4	5	6	7	× 5
	5	10	15	20	25	30	35	

If an egg costs 5 cents, two eggs will cost 10 cents and 3 eggs will cost 15 cents.

In table 2 we can see that the top series are multiplied by 5 to calculate the bottom series.

Two series of numbers are proportional when there is an operator which multiplies or divides that transforms one series into the other.



However, there are related amounts which are not proportional. For example, the age of a girl and her weight. If a six-year-old girl weighs 20 kilos, a twelve-year-old girl (6×2) will not necessarily weigh 40 kilos (20×2). When she is 24 years old (6×4) she will not necessarily weigh 80 kilos. (20×4).

EXERCISE 8.

Say if these are related or proportional amounts:

1. The kilos of rice I sell and the money they pay me are...	<i>Related</i>	<i>Proportional</i>
2. The age of a boy and his weight are...	<i>Related</i>	<i>Proportional</i>
3. The fertilizer we put in a field and the harvest we get are...	<i>Related</i>	<i>Proportional</i>
4. The number of carpenters and the number of chairs they make are...	<i>Related</i>	<i>Proportional</i>

Direct or inverse proportions.

The proportions above were direct, that is to say, when one amount gets bigger, the other get bigger, too.

Now let's see some inverse proportions, that is, when one amount gets bigger, the other gets smaller.

For example: the age of an old man and the strength he has. When his age gets greater, his strength gets smaller. It is an inverse proportion.

EXERCISE 9. Say if these are direct or inverse proportions:

1. The speed of a train and the time it takes to go from one city to another are in ... proportion.	<i>Direct</i>	<i>Inverse</i>
2. The number of carpenters and the number of tables they make are in ... proportion.	<i>Direct</i>	<i>Inverse</i>
3. The power of a car and its speed are in ... proportion.	<i>Direct</i>	<i>Inverse</i>
4. The time it takes to make a road and the number of workers are in ... proportion.	<i>Direct</i>	<i>Inverse</i>
5. The number of couples who get married and the number of children who are born are in ... proportion.	<i>Direct</i>	<i>Inverse</i>
6. The size of the hole in a barrel and the time it takes to empty are in ... proportion.	<i>Direct</i>	<i>Inverse</i>



Directly proportional and inversely proportional amounts.

When the ratio between two amounts is the same, they are a proportion.

They are directly proportional if, for example, when there is three times as much of one thing, there is three times as much of the other.

Example: a pack of cigarettes costs 2 euros; 3 packs will cost 6 euros (2×3).

They are inversely proportional when if there is 7 times as much of one thing, there is one 7th of the other.

Example: If a reaper (=segador) takes 21 hours to mow (=segar) a field, 7 reapers will take 3 hours. ($21 : 7$).

EXERCISE 10. Say if these proportions are direct or inverse:

1. The number of woodcutters and the number of trees they can cut down is...	<i>directly proportional</i> <i>inversely proportional</i>
2. The speed of an airplane and the time it takes to make a trip are...	<i>directly proportional</i> <i>inversely proportional</i>
3. The amount of cigarettes I smoke and the amount of money I spend are...	<i>directly proportional</i> <i>inversely proportional</i>
4. The number of notebooks I buy and the amount of money I have to pay are...	<i>directly proportional</i> <i>inversely proportional</i>
5. If you have 12 to buy books, the number of books you can buy and their price are...	<i>directly proportional</i> <i>inversely proportional</i>
6. The number of painters and the time they take to paint a house are...	<i>directly proportional</i> <i>inversely proportional</i>



HOW TO SOLVE A PROBLEM WITH A DIRECT PROPORTION**(Regla de tres directa)**

La **regla de tres directa** es un procedimiento que tiene por objeto hallar el cuarto proporcional. Se plantea de la siguiente forma:

a) Se colocan los datos y se determina si la proporcionalidad es directa:

<u>Magnitud A (unidad)</u>	(D)	<u>Magnitud B (unidad)</u>
a	—————→	c
b	—————→	x

b) Se forma la proporción y se calcula el cuarto proporcional:

$$\frac{a}{b} = \frac{c}{x} \Rightarrow x = \frac{b \cdot c}{a}$$

EXERCISE 11. If 10 calculators cost £120, how much will 8 calculators cost?

EXERCISE 12. Questions for practice.

Work out the answer to each of these questions:

a) If 5 bags of sweets contain 90 sweets in total, calculate how many sweets seven bags will contain.

b) If 25 floppy disks cost £5.50, calculate the cost of 11 floppy disks.

EXERCISE 13. Work out the answers to the questions below and fill in the boxes.

If 5 tickets for a play cost £40, calculate the cost of:

a) 6 tickets _____

b) 9 tickets _____

c) 20 tickets _____

EXERCISE 14. To make 3 glasses of orange squash you need 600ml of water. Work out how much water you need to make:

a) 5 glasses of squash _____ ml

b) 7 glasses of squash _____ ml

c) 1 glass of squash _____ ml



EXERCISE 15. If 10 litres of petrol cost £8.20, calculate the cost of:

a) 4 litres _____

b) 12 litres _____

c) 30 litres _____

EXERCISE 16. A baker uses 1800 grams of flour to make 3 loaves of bread. How much flour will he need to use if he wants to make:

a) 2 loaves _____ grams

b) 7 loaves _____ grams

c) 24 loaves _____ grams

EXERCISE 17. Ben buys 21 football stickers for 84p. Calculate the cost of:

a) 7 stickers _____ pence

b) 12 stickers _____ pence

c) 50 stickers _____ pence

EXERCISE 18. 16 teams, each with the same number of people, enter a quiz. At the semi-final stage there are 12 people left in the competition.

How many people entered the competition? _____ people

EXERCISE 19. Three identical coaches can carry a total of 162 passengers.

How many passengers can be carried on seven of these coaches? _____ people.

EXERCISE 20. If 3 litres of petrol cost € 3.45, calculate the cost of:

a) 5 litres

b) 23.5 litres

EXERCISE 21. If we travel 136 km in 1.5 hours driving at a constant speed,

- a) How many km can we travel in 7.4 h?

- b) How many hours do we need to travel 200 km?

EXERCISE 22. Adrian finds that in each delivery of 500 bricks there are 20 broken bricks. How many bricks are broken in a delivery of 7,500?



EXERCISE 23. In a drink 53 ml of fruit are mixed with 250 ml of water. How many litres of water are there in 30 l of that drink?

EXERCISE 24. A car uses 25 litres of petrol to travel 176 miles. How far can the car travel using 44 litres of petrol?



3. INVERSE PROPORTION.

Two values are in inverse proportion when one value increases as the other value decreases.

We say that there is an inverse proportionality between two magnitudes if when increasing one magnitude, (double, triple...) the other decreases (half, third...), and when reducing one (half, third...), the other increases (double, triple...).

Example:

Nº de obreros	Nº de días
10	6
20	3
30	2
40	1,5
50	1,2
60	1

HOW TO SOLVE A PROBLEM WITH AN INVERSE PROPORTION: (REGLA DE TRES INVERSA)

Se aplica el siguiente procedimiento:

a) Se colocan los datos y se determina si la proporcionalidad es inversa:

<u>Magnitud A (unidad)</u>	(I)	<u>Magnitud B (unidad)</u>
a	→	c
b	→	x

b) Se forma la proporción en la que la razón de las cantidades de la magnitud A aparece invertida:

$$\frac{b}{a} = \frac{c}{x} \Rightarrow x = \frac{a \cdot c}{b}$$



EXERCISE 25. A book has 300 pages, and each page has 28 lines. If we need the book to be only 280 pages long, how many lines must a page contain?

EXERCISE 26. A farmer has enough grain to feed 50 cows for 10 days. He sells 10 cows. How many days will the grain last now?

EXERCISE 27. In an army camp, there are 1,200 soldiers and they have enough food for 8 weeks. Then 300 more soldiers join the camp. How many weeks will the food last now?

EXERCISE 28. We know that the current (A) in an electric circuit is inversely proportional to the resistance (R) in the circuit. When the resistance is 3 ohms, the current is 2 amperes. Find the resistance if the current is 5 amperes; and find the current when the resistance is 4 ohms.



EXERCISE 29. Some people working at a rate of 6 hours a day can complete the work in $19 \frac{1}{2}$ days. They receive another contract, so they want to finish their work quickly. Now they start working $6 \frac{1}{2}$ hours a day. How many days will they take to finish their work?

EXERCISE 30. 35 trucks are needed to transport all the grain in a warehouse at a rate of 80 bags a truck. But the truck drivers object, saying it is too heavy, and they want to carry only 70 bags a truck. How many more trucks are needed to transport the grain?

EXERCISE 31. A truck that carries 3 tons needs 15 trips to carry a certain amount of sand. How many trips are needed to carry the same amount of sand with another truck that carries 5 tons?



EXERCISE 32. An automobile factory produces 8,100 vehicles in 60 days. With the production rhythm unchanged, how many units will be made in one year?

EXERCISE 33. A driver takes $3\frac{1}{2}$ hours to drive 329 km. How long will it take him to make another trip in similar conditions as the previous one, but travelling 282 km?

EXERCISE 34. Two hydraulic shovels make the trench for a telephone cable in ten days. How long will it take to make the trench with 5 shovels?



4. PERCENTAGE.

4.1. What does % mean?

You probably already know a bit about percentages. Shops use percentages in sales. Banks use them for loan rates. Weather forecasts use them to tell us the probability of rain. But what does **percentage** mean?

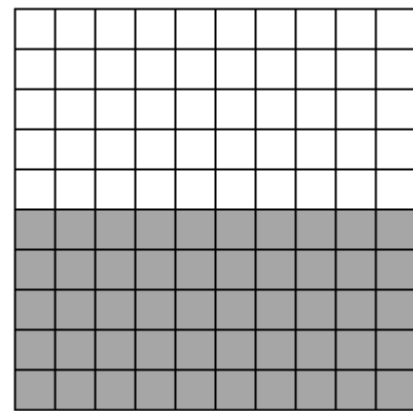
'per cent' means 'out of every 100'



Look at this square. It has been divided into 100 equal parts. 50 parts are shaded grey.

We say that fifty per cent of the square is shaded grey.

You can use a percentage symbol to write this as 50%.



Now 20 parts have been coloured black. 20 out of the 100 is 20%, so 20% of the square is black.

There are 30 parts not shaded. 30 out of 100 is 30%, so 30% of the square is white.

What happens if you add up the percentages for the grey, black and white parts?

$$50\% + 20\% + 30\% = 100\%$$

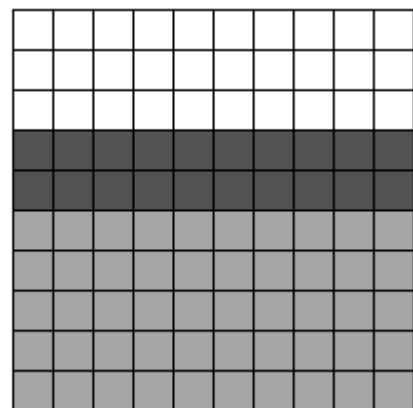
So the whole square is equal to 100%

100% is '100 out of 100'. It means full marks in a test, or 100% natural means no artificial additives. If you give 100%, you work really hard to do everything you can.

30

20

50



4.2. Fractions and %

Skirts are on sale with 50% off the original price of £20. How much are they now?

If you said £10, well done! It is quite easy to find the answer if you can remember that finding 50% is the same as finding a half, or dividing by two.

Here are the fractions for some other percentages:

percentage	fraction
10%	$\frac{1}{10}$
20%	$\frac{1}{5}$
25%	$\frac{1}{4}$
50%	$\frac{1}{2}$
75%	$\frac{3}{4}$

Examples



20% is the same as $\frac{1}{5}$ so divide by 5 to find the saving.

10% is the same as $\frac{1}{10}$ so divide by 10 to find the saving.



Example

To pass a national exam you need to get 75%. There are 40 questions altogether. How many do you need to get right?

75% is the same as $\frac{3}{4}$ so find $\frac{1}{4}$ then multiply by 3 to find $\frac{3}{4}$.

$$\frac{1}{4} \text{ of } 40 = 40 \div 4 = 10$$

$$\frac{3}{4} \text{ of } 40 = 3 \times 10 = 30$$

So you need to get **30** questions right to pass the exam.

A percent is a ratio of a number to 100.

A percent is expressed using the symbol %.

A percent is also equivalent to a fraction with denominator 100.



4.3. Percent as a decimal

Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right.

Examples: $0.15 = 15 \text{ hundredths} = 15 \%$

$$0.0006 = 0.06 \%$$

Converting from percent to decimal form is similar, only you move the decimal point 2 places to the left.

Examples:

- Express 3% in decimal form. Moving the decimal 2 to the left (and adding in 0's to the left of the 3 as place holders) we get 0.03.
- Express 97.5% in decimal form. We move the decimal 2 places to the left to get 0.975.

Exercise 35. Convert the following percentages to decimals.

- | | | |
|--------|----------|---------|
| a) 75% | b) 1.74% | c) 3.7% |
| d) 80% | e) 15% | f) 0.6% |
| g) 9% | h) 0.07% | |

Exercise 36. Convert the following decimals to percentages.

- | | | |
|---------|---------|---------|
| a) 0.5 | b) 0.74 | c) 0.35 |
| d) 0.08 | e) 0.1 | f) 0.52 |
| g) 0.8 | h) 0.07 | i) 0.04 |
| j) 0.18 | k) 0.4 | l) 0.3 |

5. CALCULATIONS WITH PERCENTAGES.

5.1. Percentage of a quantity

To calculate the percentage of a quantity we must multiply it by the percent and divide by 100.

Example: Calculate the 5% of 72 5% of 72 is $\frac{5 \cdot 72}{100} = 3.6$

Exercise 37. Calculate:

- a) 15% of 540
- b) 3% of 320
- c) 5.3% of 7
- d) 6% of 5430

5.2. Calculate the number when we know the Percentage

We must multiply the percentage by 100 and divide by the percent.

Example: The 22% of a number is 66, which is the number?

The quantity is $\frac{66 \cdot 100}{22} = 300$

Exercise 38. Calculate the number if:

- a) 95% of the number is 1,020
- b) 5% of the number is 7

c) 15.3% of the number is 500

d) 83% of the number is 1,086

5.3. Express a ratio or a fraction as a percent

To write as a percent a ratio or a fraction, we may convert them into a number dividing and then multiply by 100

Example: Calculate 27 out of 32 as a percent. We do $\frac{27}{32} \cdot 100 = 84.3\%$

Exercise 39. Express each of the following as percentages.

a) 8 out of 50

b) 3 out of 25

c) 8 out of 20

d) 3 out of 10

e) 6 out of 50

f) 6 out of 40

g) 12 out of 80

h) 84 out of 200

i) $\frac{23}{75}$

j) $\frac{3}{5}$

k) $\frac{102}{37}$

5.4. Calculate a number increased with percentage

We add to the number the percentage.

Example: Calculate the value of 320 increased by a 5%

1. The increase is 5% of 320 is $\frac{320 \cdot 5}{100} = 16$
2. The final value is $320 + 16 = 336$

5.5. Calculate a number decreased with a percentage

We subtract to the number the percentage.

Example: Calculate the value of 320 decreased by a 12%

1. The decrease is 12% of 320 is $\frac{320 \cdot 12}{100} = 38.4$
2. The final value is $320 - 38.4 = 281.6$

5.6. Percentage increase decrease

Percent increase and decrease of a value measure how that value changes, as a percentage of its original value.

Percentage *increases and decreases* are calculated using:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \times 100$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \times 100$$

Examples:

1. A collectors' comic book is worth €120 in 2004, and in 2005 its value is €132. The increase in price is €12; 12 is 10% of 120, so we say its value increased by 10%.
2. Amy is training for the 1500 metre race. When she started training she could run 1,500 metres in 5 minutes and 50 seconds. After a year of practice her time decreased by 8%. How fast can she run the race?

Her previous time was $5 \times 60 + 50 = 350$ seconds, and 8% of 350 is 28, so she can now run the race in $350 - 28 = 322$ seconds (5 minutes and 22 seconds).

3. A compact disc that sells for €12 is on sale at a 20% discount. How much does the disc cost on sale?

The amount of the discount is 20% of €12, which is €2.40, so the sale price is $€12.00 - €2.40 = €9.60$.

Exercise 40. A magazine for teenagers sells 110,000 copies each month. The company's president wants to increase the sales by 6%. How many extra magazines would they have to sell to reach this goal?

Exercise 41. Chocolate bars normally cost 80 cents each, but they are on sale for 40 cents each. What percent discount is this?

Exercise 42. Cinema tickets sell for €8.00 each, but if you buy 4 or more you get €1.00 off each ticket. What percent discount is this?



Exercise 43. A firm decides to give 20% extra soap powder for free in their packets. How much extra soap powder would be given away free with packets which normally contain...

- a) ... 2 kg of soap powder?

- b) ... 1.2 kg of soap powder?

Exercise 44: A house costs €30,000. A buyer is given a 10% discount. How much money does the buyer save?

Exercise 45. Karen bought an antique vase for €120. Two years later its value has increased by 25%. What is the new value of the vase?

Exercise 46. When Wendy walks to school she covers a distance of 1,800 m . One day she discovers a shortcut, which reduces this distance by 20%. How much shorter is the new route?

Exercise 47.: Kim receives €2.00 per week, and her mother decides to increase her pocket money by 40%. How much extra does Kim receive every week now?



Here are **two** activities to see what percentages look like. In the second one you can compare percentages and fractions.

Percentages of something.

<http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/percentages/introduction/flash2.shtml>



See percentages and fractions side by side.

<http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/percentages/introduction/flash3.shtml>



Do the following interactive exercises to learn how to calculate percentages.

Exercise 1.

http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/txc/txc_p.html

Exercise 2

http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/proporcionalidad/txcc/txcc_p.html

Exercise 3. Calculate 30% of 1,000 and 20% de 300.

Watch this **video** (http://www.mathplayground.com/howto_perfracdec.html)



Do the following tests to check what you have learnt about percentages.

(QUIZ Percentages Start)

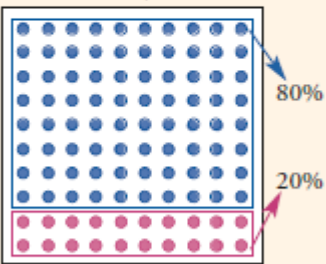
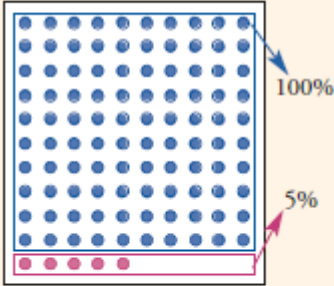
Test 1: Very easy (LEVEL A) <http://www.bbc.co.uk/skillswise/quiz/ma16perc-e3-quiz>

Test 2: A bit harder (LEVEL B) <http://www.bbc.co.uk/skillswise/quiz/ma16perc-e3-quiz>

Test 3: Only for confident students (LEVEL C)

<http://www.bbc.co.uk/skillswise/quiz/ma16perc-e3-quiz>

REMEMBER THAT:

<p>Disminución porcentual</p> <p>Disminuir una cantidad en un 20% es lo mismo que calcular el $100\% - 20\% = 80\%$ de la cantidad</p> <p>Disminución porcentual</p> <p>Disminuir una cantidad en un 20% es lo mismo que calcular el $100\% - 20\% = 80\%$ de la cantidad.</p>  <p style="text-align: center;">$80\% = \frac{80}{100} = 0,8$</p>	<p>Aumento porcentual</p> <p>Aumentar una cantidad un 5% es lo mismo que calcular el $100\% + 5\% = 105\%$ de la cantidad</p> <p>Aumento porcentual</p> <p>Aumentar una cantidad en un 5% es lo mismo que calcular el $100\% + 5\% = 105\%$ de la cantidad.</p>  <p style="text-align: center;">$105\% = \frac{105}{100} = 1,05$</p>
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EXTRA EXERCISES

Exercise 1. The Price of a bar of chocolate is 35 c and includes 7 c profit. Express the profit. Express the profit as a percentage of the price.



Exercise 2. In the crowd at a football match there were 35,000 Manchester United supporters and 25,000 Manchester City supporters. What percentage of the crowd supported each team?

Exercise 3. A school won a prize of €1,800. The Headmaster spent €750 on a new computer and the rest on software. What percentage of the money was spent on software?

Exercise 4. James needs another 40 football stickers to complete his collection. There are a total of 500 stickers in the collection. What percentage of the collection does he have?



Exercise 5. A 750 ml bottle of shampoo contains 200 ml of free shampoo. What percentage is free?

Exercise 6. Adrian finds that in a delivery of 1,500 bricks there are 50 broken. What is the percentage of broken bricks?

Exercise 7. A glass of drink contains 50 ml of fruit juice and 200 ml of lemonade. What percentage of the drink is fruit juice?

Exercise 8. Find each of the following, rounding your answers to the nearest penny.

- a) 30% of 150€
- b) 12% of 903€
- c) 12.6% of 140€
- d) 4.5% of 320€
- e) 5.9% of 50€
- f) 8.2% of 18€

Exercise 9. An MP4 player has a normal price of €150.

a) On a sale its normal price is reduced by 12%. Find the sale price.

b) After the sale, normal prices are increased by 12%. Find the new price of the MP4 player.



Exercise 10. Peter earns €9,000 per year. He does not pay taxes on the first €3,500 he earns and pays 25% taxes on the rest. How much tax does he pay?

Exercise 11. A new gas supplier offers a 25% discount on the normal price and a further 5% discount if customers pay online. For one customer the gas bill is €130. Find out how much they have to pay after both discounts.

REMEMBER	
$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \times 100$	$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \times 100$

Exercise 12. A baby weighed 5.6 kg and six weeks later her weight increases to 6.8 kg. Find the percentage increase.

Exercise 13. A factory produces blank DVDs at a cost of €0.88 and sells them for €1.10. Find the percentage profit.

Exercise 14. A new car cost €11,500 and one year later it is sold for €9,995. Find the percentage reduction in the value of the car.



Exercise 15. An investor bought some shares at a price of €14.88 each. The price of the shares drops to €11.45. Find the percentage loss.

Exercise 16. A supermarket offers an €8 discount to all customers spending more than €40. Karen spends €42.62 and John spends €78.82. Find the percentage saving for each one.

Exercise 17. In a year, the value of a house increases from €146,000 to €148,000. Find the percentage increase in the value of the house.

Exercise 18. A battery powers an iPod for 12 hours. An improved version of the battery powers the iPod for an extra 30 minutes. Find the percentage increase in the life of the batteries.

