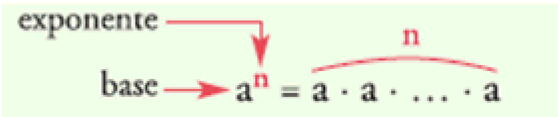
Unit 6. POWERS AND ROOTS.

1. POWERS.

Complete the following table:

1	2	3	4	5	6	7	8	9	10
1	4					49			

A **power** is a multiplication with identical factors:



In English: base → ………………………………. Exponente →…………………………

If the index (exponent) is 2, we read it "squared".

E.g. 8^2 = ocho al cuadrado = eight squared

If the index (exponent) is 3, we read it "cubed".

E.g. 7^3 = siete al cubo = seven cubed

If the index (exponent) is 4, we read it "to the fourth power", or "to the power of four".

E.g. 2^4 = dos a la cuarta potencia = two to the fourth power.

Other examples:

- 9^3 = nine cubed
- 20^4 = twenty to the fourth power
- 45^5 = forty-five to the power of five / forty-five to the fifth power

SIGN OF A POWER.

Read and think about the following table:

Base	Exponent	Sign of the result	Examples
+	even or odd	+	$2^3 = 8$ $2^4 = 16$
–	even	+	$(-2)^2 = 4$
–	odd	–	$(-2)^5 = -32$

Exercises:

$(-2)^3 =$

$(-3)^3 =$

$(-5)^4 =$

$(-10)^3 =$

$(7)^3 =$

f) $2^3 =$

g) $(-1)^4 =$

h) $(-5)^3 =$

i) $(-10)^6 =$

j) $(-7)^3 =$

USE THE CALCULATOR.

There are some very useful keys to do powers:

$\boxed{x^2}$ → **squared (elevar al cuadrado) E.g.** 17^2 $\boxed{17}\boxed{x^2}\boxed{=}$ **289**

$\boxed{x^3}$ → **cubed (elevar al cubo) E.g.** 14^3 $\boxed{14}\boxed{x^3}\boxed{=}$ **2744**

$\boxed{\wedge}$ **or** $\boxed{x^y}$ → **x powered y . E.g.** $\boxed{5}\boxed{\wedge}\boxed{6}\boxed{=}$ **15625**

$\boxed{\text{EXP}}$ → **10 powered … E.g.** $\boxed{\text{EXP}}\boxed{5}\boxed{=}$ **100000**

Exercise: Calculate with the calculator:

a) $(-6)^2 =$

b) $5^3 =$

c) $(2)^{20} =$

d) $(10)^8 =$

e) $(-6)^{12} =$

2. SCIENTIFIC NOTATION.

When we use scientific notation to write a number, we write the number as the product of a number with only one unit by a power of 10. For example, 123 is 1.23×10^2. In Spanish:

La **notación científica** de un número es la expresión de dicho número como producto de un número decimal en el que la parte entera está formada por una sola cifra no nula y una potencia entera de 10

We use scientific notation for very big numbers or very small numbers:

1 light year = $9.461 \cdot 10^{12}$ km = 9,461,000,000,000 km

Radius of an oxygen atom = $6.6 \cdot 10^{-11}$ m = 0.000000000066 m

Examples:

234 000 000 000 000 000 000 = ………………………………………………

876 000 000 000 000 000 = ………………………………………………………………

0.000 000 000 000 154 = ……………………………………………………….

0.000 000 000 000 000 000 0023 = ………………………………………………

3. PROPERTIES OF POWERS.

- ✓ **Multiplication of powers with the same base** *(=Multiplicación de potencias de la misma base)*
- ✓

The product of two powers with the same base is another power which has the same base, and an exponent which is the addition of the exponents of the two powers: $a^n \cdot a^p = a^{n+p}$

(In Spanish: El producto de dos potencias de la misma base es otra potencia que tiene la misma base y como exponente la suma de los exponentes).

E.g: $5^3 \cdot 5^4 = 5^7$

Checking:

E.g.: $3^2 \cdot 3^4 = 3^6$

Checking:

- ✓ **Division of powers with the same base**

The division of two powers with the same base is another power which has the same base, and an exponent which is the difference between the exponents of the two powers: $a^n : a^p = a^{n-p}$

In Spanish: El cociente de dos potencias de la misma base es otra potencia que tiene la misma base y como exponente la diferencia de los exponentes.

E.g: $5^7 : 5^3 = 5^4$

Checking:

$$5^7 : 5^3 = \frac{5^7}{5^3} = \frac{\cancel{5} \cdot \cancel{5} \cdot \cancel{5} \cdot 5 \cdot 5 \cdot 5 \cdot 5}{\cancel{5} \cdot \cancel{5} \cdot \cancel{5}} = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$

E.g.: $3^5 : 3^4 = 3^1 = 3$

Checking:

✓ **Power of a power**

The power of a power is another power which has the same base, and an exponent which is the product of the exponents of the two powers: $(a^n)^p = a^{n \cdot p}$

In Spanish: La potencia de una potencia es otra potencia que tiene la misma base y como exponente el producto de los exponentes.

E.g.: $(5^2)^3 = 5^6$ **Checking:**

$$(5^2)^3 = \overbrace{(5^2) \cdot (5^2) \cdot (5^2)}^{3} = \overbrace{(5 \cdot 5)}^{2} \cdot \overbrace{(5 \cdot 5)}^{2} \cdot \overbrace{(5 \cdot 5)}^{2} =$$
$$= 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^6$$

E.g. $(3^5)^2 = 3^{10}$

Checking: $(3^5)^2 = 3^5 \cdot 3^5 = (3{\cdot}3{\cdot}3{\cdot}33) \cdot (3{\cdot}3{\cdot}3{\cdot}3{\cdot}3) = 3^{10}$

✓ **Power of a multiplication**

$(a \cdot b)^n = a^n \cdot b^n$

E.g. $(5 \cdot 7)^3 = 5^3 \cdot 7^3$

E.g. $(3 \cdot 5)^4 = 3^4 \cdot 5^4$

✓ **Power of a division**

$(a : b)^n = a^n : b^n$

E.g.: $(3 : 5)^3 = 3^3 : 5^3 = \frac{27}{125}$

✓ **Remember this:** $a^0 = 1$, **so any number to the zero power is equal to 1.**

E.g. $5^0 = 1$, $2^0 = 1$, $(0.5)^0 = 1$, $(-5)^0 = 1$

Checking: Applying the rule for division of powers with the same base:
$a^n : a^p = a^{n-p}$, we can see that $a^n : a^n = a^{n-n} = a^0$.
Any number divided by itself is equal to 1, so any power (a^n) divided by itself ($a^n : a^n = a^0$) is equal to 1.

4. FIND THE ERRORS.

The most usual errors with powers are in the following examples, find them:

$2^3 = 6$?

$3^0 = 0$?

$-2^2 = -4$?

$(2+3)^2 = 2^2 + 3^2$?

$(3-1)^2 = 3^2 - 1^2$?

Homework. Page 109, exercises: 12, 13, 14, 15, 16, 17, 18 (write the instructions)

EXERCISES

UNIT 6. POWERS AND ROOTS.

1. Look at this example: $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$
   Fill and copy.

   a) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \ldots = \ldots$

   b) $7 \cdot 7 \cdot 7 \cdot 7 = \ldots = \ldots$

   c) $\qquad = 5^3 = \ldots$

   d) $\qquad = 8^5 = \ldots$

2. Calculate in your mind.

a) 2^4 b) 6^3 c) 3^5 d) 20^4 e) 30^0

3. Calculate doing the operations here

a) 5^5 b) 9^5 c) 1^{10} d) 15^3 e) 16^4

4. Calculate with the calculator:

a) 4^{12} b) 5^{10} c) 45^3 d) 67^4 e) 99^3

5.Write the solutions:

a) 10^2 b) 10^6 c) 10^{10} d) 10^{12} e) 10^{16}

6.Write as a power:
a) One hundred _______________
b) One hundred thousand _________________
c) One hundred thousand billion __________________________________
d) One hundred million ____

7.Arrange from the smallest to the biggest.

$8 \cdot 10^9$	$17 \cdot 10^7$	$98 \cdot 10^6$	10^{10}	$16 \cdot 10^8$	$9 \cdot 10^9$

8. Calculate using the properties of powers:

a) $5^4 \cdot 2^4$ b) $4^3 \cdot 5^3$ c) $2^6 \cdot 5^6$
d) $6^3 \cdot 5^3$ e) $8^2 \cdot 5^2$ f) $25^3 \cdot 4^3$
g) $4^6 : 2^6$ h) $6^5 : 3^5$ i) $8^4 : 4^4$
j) $15^3 : 5^3$ k) $20^4 : 5^4$ l) $18^2 : 9^2$

9. Calculate using the properties of powers:
(Remember that:
☞ $a^n \cdot a^m = a^{m+n}$ $a^m : a^n = a^{m-n}$ $(a^m)^n = a^{m \cdot n}$

a) $8^2 \cdot 8^4$ b) $2^5 \cdot 2^7$ c) $10^2 \cdot 10^2$
d) $x^8 \cdot x^3$ e) $a^5 \cdot a^5$ f) $k^7 \cdot k^6$
g) $5^{10} : 5^6$ h) $3^{12} : 3^4$ i) $12^{10} : 12^9$
j) $x^7 : x^5$ k) $a^9 : a^2$ l) $k^{12} : k^{12}$
m) $(2^5)^2$ n) $(7^4)^3$ ñ) $(8^2)^2$
o) $(x^3)^2$ p) $(a^5)^3$ q) $(k^4)^4$

10. Calculate and compare:

a) $(1+4)^3$ ______ $1^3 + 4^3$ ______ b) $(1+4)^4$ ______ $1^4 + 4^4$ ______

5. ROOTS.

5.1 SQUARE ROOT. Fill in the table:

Number	1	2	3	4	5	6	7
Perfect Square (Cuadrado Perfecto)	1	4					

We usually write:

$\sqrt{4} = 2$ $\sqrt{1} = 1$ $\sqrt{9} = 3$

But this is not absolutely true, pay attention:

$\sqrt{a} = b$ si $b^2 = a$

$\sqrt{\ }$	Signo radical
a	Radicando
b	Raíz

and then, look at this:

$\sqrt{4} = \pm 2$ because $2^2 = 4$ and $(-2)^2 = 4$

$\sqrt{9} = \pm 3$ because $3^2 = 9$ and $(-3)^2 = 9$

$\sqrt{1} = \pm 1$ *because* $1^2 = 1$ *and* $(-1)^2 = 1$

$\sqrt{0} = 0$

$\sqrt{-9} = no\ existe = \nexists$

So, a number can have two square roots, one or none. In Spanish:

Un número puede tener dos raíces cuadradas, una o ninguna.

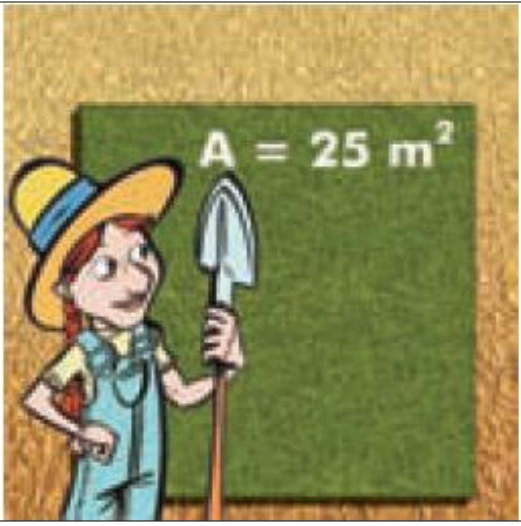
Radicando	Número de raíces	Ejemplo	Comprobación
0	Una	$\sqrt{0} = 0$	$0^2 = 0$
+	Dos opuestas	$\sqrt{9} = \pm 3$	$3^2 = 9$; $(-3)^2 = 9$
–	Ninguna	$\sqrt{-9}$ no existe	

E.g.: How many roots has $\sqrt{4}$ got? Two roots, they are 2 and -2, because $2^2 = 4$ and $(-2)^2 = 4$.

E.g.: How many roots has $\sqrt{-16}$ got? None, or no roots.

E.g.: How many roots has $\sqrt{0}$ got ? One root, and it is 0, because $0^2 = 0$.
E.g.: How many roots has $\sqrt{81}$ got? Two roots, they are 9 and -9.

PROBLEM:

	***A gardener has a 25 m² square garden. How many metres long is the side?*** ***Translation:***

LET'S APPROXIMATE ROOTS:

a) $5 < \sqrt{27} < 6$ because $5^2 = 25$ and $6^2 = 36$.

b)............ $< \sqrt{10} <$............... because ....

c)............ $< \sqrt{17} <$............... because ....

d)............ $< \sqrt{70} <$............... because ....

5.2. CUBE ROOT. (Raíz cúbica)

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[3]{27} = 3$ because $3^3 = 27$

$\sqrt[3]{1} = 1$ because $1^3 = 1$

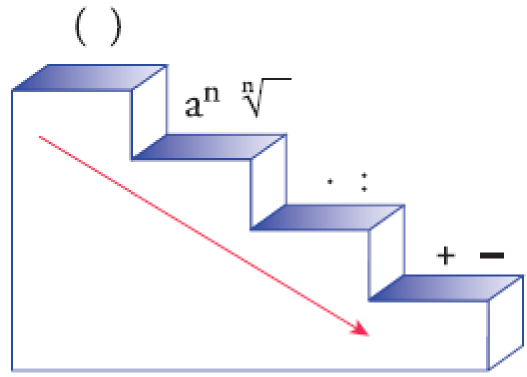
$\sqrt[3]{125} =$ because.........................

$\sqrt[3]{0} =$ because............................

$\sqrt[3]{-8} =$ because............................

OPERATIONS PRIORITY.

Remember this:

	Brackets. Powers and roots. Multiplications and divisions. Additions and subtractions.

Example:

$$(75 - 8^2) \cdot \sqrt{36} = (75 - 64) \cdot 6 = 11 \cdot 6 = 66$$

EXERCISES

UNIT 6. POWERS AND ROOTS.

Exercise 1. Revise the example and fill the gaps.

- $8^2 = 64 \leftrightarrow \sqrt{64} = 8$

a) $\square^2 = 36 \leftrightarrow \sqrt{36} = \square$ b) $\square^2 = 256 \leftrightarrow \sqrt{256} = \square$

Exercise 2. Calculate the value of "m" in each case.

a) $\sqrt{m} = 8$ b) $\sqrt{m} = 20$ c) $\sqrt{m} = 45$

Exercise 3. Calculate the value of "a" in each case.

a) $a^2 = 81$ b) $a^2 = 100$ c) $a^2 = 441$

Exercise 4. Calculate in your mind the estimated value of:

a) $\sqrt{90}$ b) $\sqrt{121}$ c) $\sqrt{1\,785}$

Exercise 5. Use the calculator to work out:

a) $\sqrt{655}$ b) $\sqrt{1\,024}$ c) $\sqrt{1\,369}$ d) $\sqrt{4\,225}$

Exercise 6. Check whether all of the following numbers are perfect squares.

1 936 6 556 8 464 16 076 11 025 178 929

Exercise 7.

¿Cuántas losas de un metro cuadrado se necesitan para cubrir un patio cuadrado de 22 m de lado?

Exercise 8.

Una finca cuadrada tiene una superficie de 900 metros cuadrados. Calcula la longitud de su lado.

Exercise 9.

Calcula el número de cubitos de arista unidad que caben en un cubo de arista 10 unidades.

Exercise 10.

Se ha enlosado una habitación cuadrada con 2 209 baldosas, también cuadradas. ¿Cuántas filas forman las baldosas?

Exercise 11. **Calculate using the priority of operations:**

a) $\left(7\sqrt{36} - 8^2 + 15\right) \cdot \sqrt{100}$ ....

b) $\left(7^2 + 476 - \sqrt{64} + 2^5\right) : \sqrt{81}$ ....

Exercise 12. Calculate:

Calculate:

a) $\sqrt{9+16}$ .... b) $\sqrt{9} + \sqrt{16}$ .... c) $\sqrt{100-64}$ .... d) $\sqrt{100} - \sqrt{64}$ ....

Exercise 13. **Calculate using the calculator:**

a) $\left(13\sqrt{81} - 12^2 + 105\right) \cdot \sqrt{625}$ ....

b) $\left(7^3 - 5\,334 - \sqrt{169} + 2^7\right) : \sqrt[3]{12\,167}$ ....

Exercise 14. **Calculate using the properties of powers:**

a) $\dfrac{(a^3)^2}{a^4} =$ ....

b) $\dfrac{(2\cdot 3)^3}{2^2\cdot 3^2} =$ ....

c) $a^6 : a^4 =$ ....

d) $(a^3 \cdot a^4) : a^5 =$ ....

e) $(4\cdot 5)^3 : (4^2\cdot 5^2) =$ ....

To revise these rules of powers, you can visit this video on the Internet:

http://www.math-videos-online.com/exponents-rules.html

To revise how to simplify roots, you can visit this video on the Internet:

http://www.math-videos-online.com/simplifying-square-roots.html