



UNIT 3. INTEGERS

1. NEGATIVE NUMBERS.

You are already familiar with some everyday number systems in which there are two distinct and opposite kinds of values. For example:

A bank account shows money paid in and money paid out; there is a zero point which is common to both kinds of number: when there is no money in the account and no money is owed to the bank. The relations 'more' or 'less', 'smaller than' or 'greater than' can be represented on either side of the zero point.

Here, you have some examples where we use negative numbers:

•Positions:	The height of Mount Everest is 8700m.	+8700 m
	A submarine which is sailing 700 m below sea level	-700 m
	The second floor of a subterranean garage	-2
•Money:	He has saved €5000 in his bank account	+€5000
	He is €400 overdrawn	-€400
•Temperatures:	The average summer temperature here is 35°C	+35°C
	The temperature in Siberia has risen 25°C below zero.	-25°C

Number 0 is not a positive or negative number.

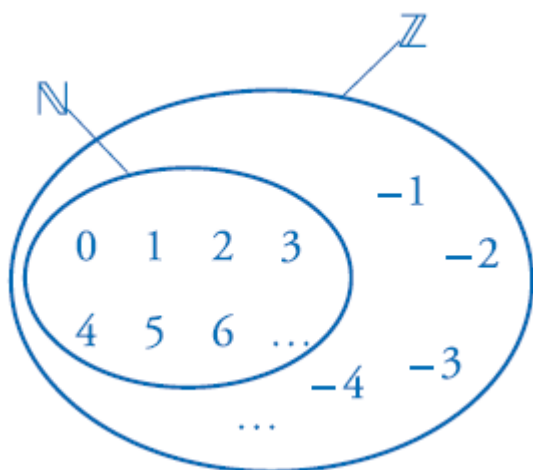
2. THE INTEGERS SET.

The integer set is formed by the natural numbers, zero and the negative numbers. It is represented by $\mathbb{Z}$.

In Spanish: _____

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$





The integer set does not have a beginning or an ending: it has infinite elements.

In this model the relation of "smaller than" $<$ means "to the left of" and "greater than" $>$ means "to the right of"

$$\begin{aligned} & \dots < -2 < -1 < 0 < 1 < 2 < 3 < \dots \\ & \dots > 3 > 2 > 1 > 0 > -1 > -2 > -3 > \dots \end{aligned}$$

Solve the following exercises:

Write with an integer the following information:

- a) The plane is flying at 9,500 m altitude
- b) The minimum temperature yesterday was 3 degrees below zero
- c) The garage is on the second basement
- d) The diver is swimming 20 metres deep
- e) Sergio owes 25 euros

Page 51, exercise 3.

- a).....
- b).....
- c).....
- d).....

You can visit this webpage and practise this exercise:

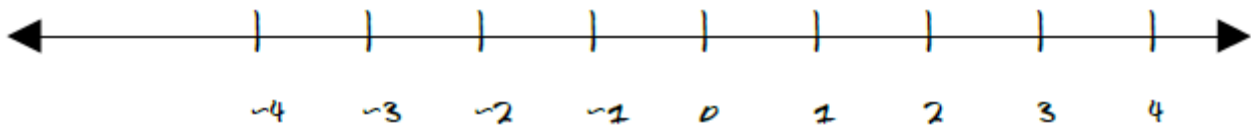
http://www.gobiernodecanarias.org/educacion/9/Usr/eltanque/todo_mate/numeros/asensor/ascensor_p.html



3. ORDER OF THE INTEGERS.

3.1. Graphical representation.

Integers can be represented on a line, like this:



3.2. Absolute value and opposite.

The **absolute value** of an integer is the number without the sign. It is represented as $|a|$ and it is called "absolute value of a". (In Spanish: *El valor absoluto de un entero es el número pero sin signo. Se representa por $|a|$ y se lee "valor absoluto de a"*)

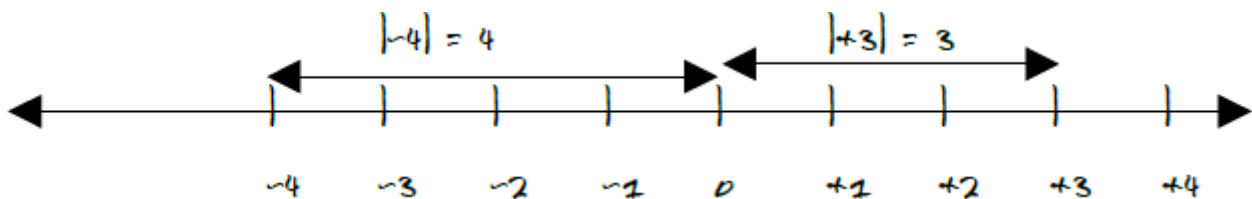
$$|+5| = 5$$

$$|-3| = 3$$

$$|-7| = 7$$

$$|+2| = 2$$

Geometrically it represents the distance from the number to zero:



The **opposite** of an integer is another integer with the same absolute value but different sign.

$$\text{Op } (+3) = -3$$

$$\text{Op } (-5) = +5$$

$$\text{Op } (-7) = +7$$

$$\text{Op } (+2) = -2$$



Solve the following exercises:

1. Work out the absolute value of these numbers:

a) $|-9| = \dots\dots$

b) $|+5| = \dots\dots$

c) $|-3| = \dots\dots$

d) $|+7| = \dots\dots$

e) $|0| = \dots\dots$

2. Find out the number that has an absolute value of 7 and is between -8 and 4.

.....



3. Work out the opposites of the following numbers:

- a) Op $(-4) = \dots\dots$ b) Op $(+8) = \dots\dots$ c) Op $(15) = \dots\dots$ d) Op $(-301) = \dots\dots$

4. Write down the absolute value of the opposites of these numbers:

- a) $|Op (+4)| = \dots\dots$ b) $|Op (-11)| = \dots\dots$ c) $|Op (-200)| = \dots\dots$ d) $|Op (+1001)| = \dots\dots$

5. Write down the opposites of the absolute value of these numbers:

- a) Op $|+4| = \dots\dots$ b) Op $|-11| = \dots\dots$ c) Op $|-200| = \dots\dots$ d) Op $|+1001| = \dots\dots$

3.3. Order in the integer set

Solve the following exercises:

1. Write down the "less than" sign ($<$) or the "greater than" sign ($>$) where it corresponds:

- a) $+4 \underline{\hspace{1cm}} +1$ b) $-1 \underline{\hspace{1cm}} -6$ c) $0 \underline{\hspace{1cm}} +3$ d) $-8 \underline{\hspace{1cm}} +2$ e) $-2 \underline{\hspace{1cm}} 0$

2. Sort out these negative numbers, with the greatest first: $-5, -1, -2, -25$

3. Sort out these numbers from the lowest to the highest: $-4, +7, -6, -3, +5$



4. OPERATIONS WITH INTEGERS.

Observa la tabla siguiente y calcula mentalmente:

-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
----	----	----	----	----	----	----	---	---	---	---	---	---	---	---

Pedro está en la casilla 2 y avanza a la derecha 3 casillas. Termina en la casilla ...

Silvia está en la casilla 6 y avanza a la izquierda 8 casillas. Termina en la casilla ...

Natalia está en la casilla -5 y avanza a la derecha 3 casillas. Termina en la casilla ...

Belén está en la casilla -1 y avanza a la izquierda 5 casillas. Termina en la casilla ...



4.1. ADDITION AND SUBTRACTION OF INTEGERS

FIRSTLY, MEMORISE THIS RULE:

$$\text{two signs in a row!} \Rightarrow \begin{cases} + \cdot + = + \\ - \cdot - = + \\ + \cdot - = - \\ - \cdot + = - \end{cases}$$



Examples: When you have two signs in a row, convert them into one using the rule above:

- a) $-(-4) = +4$
- b) $-(+2) = -2$
- c) $+(-5) = -5$
- d) $(-2) - (-4) = -2 + 4$
- e) $3 - (-5) = 3 + 5$

Addition of Signed Numbers:

- To add two integers with the same signs, you must add their absolute value and leave the same sign:

$$\begin{aligned} (+1) + (+7) &= +8 \\ (-12) + (-8) &= -20 \end{aligned}$$

- To add two integers with different signs, you must subtract their absolute values and leave the sign of the one with the greatest absolute value:

$$\begin{aligned} (+15) + (-7) &= 15 - 7 = 8 \\ (-13) + (+6) &= -13 + 6 = -7 \\ (+3) + (-4) &= 3 - 4 = -1 \\ (-2) - (+3) &= -2 - 3 = -5 \end{aligned}$$

Solve the following exercises:

1. Work out the following sums with integers:

- | | |
|------------------------------------|-----------------------------------|
| a) $-2 - 8 = \dots\dots\dots$ | b) $6 - 7 = \dots\dots\dots$ |
| c) $-19 + (-20) = \dots\dots\dots$ | d) $-10 - (-4) = \dots\dots\dots$ |
| e) $3 - (-9) = \dots\dots\dots$ | f) $16 - (-2) = \dots\dots\dots$ |

2. Work out the following combined additions:

- a) $-2 - 8 + (-3) - (-7) + (-2) - 4 = \dots\dots\dots$
- b) $6 - (+7) + (-3) - (-5) + 7 - (-8) = \dots\dots\dots$



3. Work out the following combined additions:

a) $(-3) + (-7) - (+3) - (-3) + (-5) = \dots\dots\dots$

b) $-5 + (-3) - (-2) + (-5) + (-3) = \dots\dots\dots$

c) $-(-5) + (-2) + (-3) + (-7) - (-2) = \dots\dots\dots$

d) $-(+4) - (-4) + (-3) + (-2) - (-7) = \dots\dots\dots$



4.2. MULTIPLICATION AND DIVISION

4.2.1. Multiplication of integers

To multiply two or more integers we have to multiply the signs and then the absolute value of the numbers.

To multiply the signs we use **the rule of signs**:

$$(+) \cdot (+) = (+)$$

$$(-) \cdot (+) = (-)$$

$$(+) \cdot (-) = (-)$$

$$(-) \cdot (-) = (+)$$

For example:

$$(-2) \cdot (-5) = 10;$$

$$(-3) \cdot (-5) = 15;$$

$$(-3) \cdot 7 \cdot (-1) \cdot 2 = 42$$

4.2.2. Division of integers

To divide two or more integers we have to divide the signs and then the absolute value of the numbers.

To divide the signs we use **the same rule** of signs again:

$$(+) : (+) = (+)$$

$$(-) : (+) = (-)$$

$$(+) : (-) = (-)$$

$$(-) : (-) = (+)$$

For example:

$$10 : (-5) = (-2);$$

$$(-15) : (-3) = 5$$

$$7 : (-1) = (-7)$$

$$(-10) : (-2) = 5$$

Los enteros no cumplen la ley de composición interna para la división ya que a veces la división de dos números enteros no es otro número entero. For example: $+10 : -3$



Solve the following exercise:

Work out the missing numbers:

a) $(-2) \cdot \underline{\hspace{2cm}} = -22$

b) $(-15) : \underline{\hspace{2cm}} = 15$

c) $\underline{\hspace{2cm}} \cdot (-5) = 30$

d) $\underline{\hspace{2cm}} : (+5) = -10$

e) $(+6) \cdot \underline{\hspace{2cm}} = 18$

f) $(-70) : \underline{\hspace{2cm}} = -7$

EXERCISES.**1. Calculate:**

a) $6 \cdot 5 =$

b) $-3 \cdot (-7) =$

c) $8 \cdot (-3) =$

d) $(-9) \cdot 12 =$

2. Calculate:

a) $18 : 9 =$

b) $-28 : (-2) =$

c) $15 : (-3) =$

d) $-36 : 12 =$

3. Calculate:

a) $5 (2 + 8) =$

b) $-4 (3 + 6) =$

c) $6 (7 - 4) =$

d) $-3 (5 - 3) =$

4. Calculate the value of "k" in your mind:

a) $k \cdot (-8) = -32$

b) $-12 \cdot k = 48$

c) $-63 : k = 9$

d) $k : (-4) = -16$



4.3. COMBINED OPERATIONS

4.3.1. Distributive property

Like natural numbers, integers show the distributive property, that is, the addition of two integers multiplied by another integer $(3 + 4) \cdot 6$ is equal to the addition of the third integer multiplied by each of the two first integers $(3 \cdot 6) + (4 \cdot 6)$.

Como los números naturales, los enteros satisfacen la propiedad distributiva, esto es, la adición de dos números enteros multiplicado por otro número entero es igual a la suma del tercer número multiplicado por cada uno de los dos primeros.

For example: $-2 \cdot (3 - 5) = -2 \cdot 3 - 2 \cdot (-5) = -6 + 10 = +4$.

So, if you have to multiply an integer by an addition in brackets, there are two ways of doing this:

First, the brackets and then the multiplication:

$$-5 \cdot (-3 + 5) = -5 \cdot 2 = -10$$

or applying the distributive property:

$$-5 \cdot (-3 + 5) = 15 - 25 = -10$$

Solve the following exercises:

1. Find out the result in two different ways:

a) $3 \cdot (-7 - 10) = \dots\dots\dots$

b) $(-5) \cdot (12 - 4) = \dots\dots\dots$

2. Find out the result using the distributive property:

a) $(-9) \cdot (8 - 9) = \dots\dots\dots$

b) $2 \cdot (-10 + 3) = \dots\dots\dots$

c) $4 \cdot (-5 + 9 - 6) = \dots\dots\dots$

d) $(-9 + 7 - 2) \cdot (-8) = \dots\dots\dots$

3. Complete the following exercises:

a) $(-5) \cdot (9 - 4) = -5 \cdot \dots\dots\dots = -25$

b) $\dots\dots\dots \cdot (5 - 7) = -10 + \dots\dots\dots = \dots\dots\dots$

c) $9 \cdot 8 = 9 \cdot (5 + \dots\dots\dots) = \dots\dots\dots + 27$

d) $3 \cdot (-6 + \dots\dots\dots) = 3 \cdot \dots\dots\dots + 3 \cdot (-9) = \dots\dots\dots$



4.3.2. Extract a common factor

In the operation $8 \cdot (-3) + 8 \cdot 5$ there is a factor that is repeated in both addends, which is 8. If we apply the distributive property the other way round we could write:

(in Spanish: En la suma $8 \cdot (-3) + 8 \cdot 5$ hay un factor que se repite en los dos sumandos que es el 8. Si aplicamos la propiedad distributiva pero al revés podríamos escribir)

$$8 \cdot (-3) + 8 \cdot 5 = 8 \cdot (-3 + 5)$$

Another example:

$$36 - 24 + 12 = 4 \cdot 9 + 4 \cdot (-6) + 4 \cdot 3 = 4 \cdot (9 - 6 + 3)$$

This transformation from an addition to a multiplication is called **extracting the common factor**

(in Spanish: Esta transformación de suma a producto se llama sacar factor común).

Solve the following exercises:

1. Extract a common factor of these operations and work out the result:

a) $(-5) \cdot 7 + (-5) \cdot (-12) = \dots\dots\dots$

b) $(-2) \cdot 7 + (-2) \cdot (-3) = \dots\dots\dots$

c) $5 \cdot 9 + 5 \cdot (-11) = \dots\dots\dots$

d) $(-9) \cdot (-12) + (-9) \cdot 13 = \dots\dots\dots$

e) $7 \cdot 2 + 7 \cdot (-21) = \dots\dots\dots$

f) $(-2) \cdot 7 + (-2) \cdot (-3) = \dots\dots\dots$



4.3.3. Order of the operations

When expressions have more than one operation, we have to follow some rules for the order of operations. These are the same rules as for natural numbers:

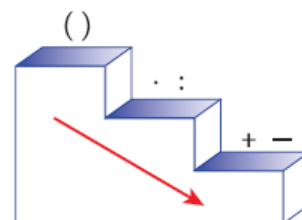
Rule 1: First perform any calculations inside the brackets.

Rule 2: Then do powers and roots.

Rule 3: Next, perform all multiplications and divisions, working from left to right.

Rule 4: Finally, perform all additions.

(In Spanish: Ahora en el paso final de la suma ya no importa el orden pero una buena idea es sumar todos los positivos, todos los negativos y después sumar al final el negativo y el positivo.)



Solved Example:

Using the order of operations, work out $(-3) - 6 \cdot (-5 + 4) : (-3) + 7$:

Step 1: $(-3) - 6 \cdot (-5 + 4) : (-3) + 7 = (-3) - 6 \cdot (-1) : (-3) + 7$ Brackets

Step 2: $(-3) - 6 \cdot (-1) : (-3) + 7 = (-3) + 6 : (-3) + 7$ Multiplication

Step 3: $(-3) + 6 : (-3) + 7 = -3 - 2 + 7$ Division

Step 4: $-3 - 2 + 7 = -5 + 7 = 2$ Addition

Solve the following exercises

1. Find out the outcome:

a) $32 + (-12) : 6 =$

b) $(-8) \times 9 - 15 \times (-3) =$

c) $(-4) \times 10 : 2 + 14 : (-7) =$

d) $27 : (-3) \times 2 - (-4) =$

e) $(-18) : 6 + 5 \times (-10) =$

f) $18 : 9 + 5 - [(-15) \times 3 + 12 \times 4] =$

g) $(-6) \times [4 - (-2)] + [-8 + (-3) \times 2] =$

h) $(-35) : (5 + 2) + (-4) \times 9 - (7 - 2 \times 5) =$

i) $[(3 - 4) + (-2)] \times 4 + 9 : (-3) \times 6 =$

j) $\{ [(-12) - (-3)] \times 8 \} + 24 : \{ [(-2) + (-6)] : 2 \} =$

YOU CAN FIND MORE INFORMATION ABOUT COMBINED OPERATIONS ON THE INTERNET, THIS IS A VERY USEFUL AND ENJOYABLE WEBSITE. COME ON! VISIT IT!



http://webs.ono.com/paco_garces/1ESO/enteros/enteros/combin_ent.htm



EXERCISES

UNIT 3. INTEGERS.

Exercise 1. Work out:

- a) $7 + 5 =$
- b) $-3 + (-6) =$
- c) $-8 + 12 =$
- d) $9 + (-3) =$

Exercise 2. Work out:

- a) $7 - 5 =$
- b) $-8 - (-6) =$
- c) $20 - (-8) =$
- d) $7 - (-3) =$

Exercise 3. Get parenthesis out and calculate:

- a) $10 + (-5) + 5 - (-3) =$
- b) $10 + (-8) - (-12) + 4 =$
- c) $-8 + 4 - (-5 + 3) - (-2 + 6) =$
- d) $(2 - 24) - (3 + 12) - (-4 - 3) =$

Exercise 4. Work out:

- a) $23 + 14 - 7 + 8 - 12 - 1 =$
- b) $15 - 13 + 4 - 15 + 3 =$
- c) $30 - 14 - 42 + 25 + 5 =$
- d) $10 + 7 - 15 - 2 - 5 + 3 + 6 =$



Exercise 5. Get parenthesis out and calculate:

- a) $15 - 9 - 8 - [-(-9)] =$
- b) $-10 - (-7) - (-4) + 3 =$
- c) $(-6) - (-5) - [-(-3)] + 1 =$
- d) $-(-2) - [-(-7)] - (-5) - [-(-3)] =$

Exercise 6. Get parenthesis out and calculate:

- a) $20 + (-15) + 8 - (-9) =$
- b) $12 + (-7) - (-10) + 6 =$
- c) $-(-6) + 8 - (-3) - (-7) =$
- d) $-2 - (-1) - (-8) + (-7) =$

Exercise 7. Calculate:

- a) $3 + 4 - 5 + 6 - 2 - 1 =$
- b) $5 - 3 + 4 - 1 + 2 =$
- c) $3 - 5 - 4 + 7 + 1 =$
- d) $5 + 3 - 7 - 2 + 1 + 4 - 6 =$

Exercise 8. Get parenthesis out and calculate:

- a) $5 + 4 - 9 - (-8 + 5) =$
- b) $-7 + 4 - 7 - (8 - 4 + 3) =$
- c) $-6 - [2 - (1 + 5)] - (-3 + 1) =$
- d) $2 + 2 + 3 - [2 - (-3)] =$

Exercise 9. Calculate:

- a) $4 \cdot 7 =$
- b) $-5 \cdot (-6) =$
- c) $7 \cdot (-6) =$
- d) $-9 \cdot 3 =$

Exercise 10. Calculate:

- a) $3 \cdot 5 \cdot (-15) =$
- b) $-4 \cdot 5 \cdot 7 =$
- c) $3 \cdot (-4) \cdot (-20) =$
- d) $-8 \cdot (-4) \cdot (-6) =$

Exercise 11. Calculate:

- a) $16 : 4 =$
- b) $-35 : (-7) =$
- c) $45 : (-5) =$
- d) $-72 : 9 =$

Exercise 12. Calculate:

- a) $18 : 2 : 3 =$
- b) $-720 : (-10) : 9 =$
- c) $-64 : 8 : 2 =$
- d) $-120 : (-12) : (-5) =$



Exercise 13. Calculate:

a) $5(7 + 9) =$

c) $8(19 - 12) =$

b) $-6(12 + 8) =$

d) $-4(15 - 3) =$

Exercise 14. Calculate:

a) $4 + 5 \cdot 2 - 4 + 8 : 4 =$

b) $9 + 6 \cdot 3 - 5 + 12 : 6 =$

c) $-16 \cdot 4 - 18 : 9 - 3 \cdot 5 =$

d) $4 - 8 : 4 - 2 + 5 \cdot 4 =$

Exercise 15. Calculate:

a) $3 \cdot 5 - 15 + 4 + 18 : 2 =$

b) $-4 \cdot 5 + 2 - 15 : 3 =$

c) $20 - 3 \cdot 4 + 25 : 5 - 1 =$

d) $32 - 7 \cdot 4 - 5 + 30 : 5 =$

Exercise 16. Work out:

a) $16 + (-5) \cdot (+4) =$

b) $20 - (-6) \cdot (-4) =$

c) $(-2) \cdot (-5) + (+4) \cdot (-3) =$

d) $(-8) \cdot (+2) - (+5) \cdot (-4) =$

e) $10 + (-4) \cdot (+2) - (+6) =$

f) $(-5) - (+4) \cdot (-3) - (-8) =$

g) $14 - (+5) \cdot (-4) + (-6) \cdot (+3) + (-8) =$

h) $(+4) \cdot (-6) - (-15) - (+2) \cdot (-7) - (+12) =$

Exercise 17. Work out:

a) $8 + (4 - 9 + 7) \cdot 2 + 4 \cdot (3 - 8 + 4) =$

b) $4 \cdot [(+5) + (-7)] - (-3) \cdot [7 - (+3)] =$

c) $(-3) \cdot (+11) - [(-6) + (-8) - (-2)] \cdot (+2) =$



d) $(-6) \cdot [(-7) + (+3) - (7 + 6 - 14)] - (+7) \cdot (+3) = \dots\dots\dots$

Exercise 18. Alexander the Great, one of the greatest generals in history, was born in 356 B. C. and died in 323 B. C. At what age did he die? How many years have passed since then?

Exercise 19. A water park businessman does this summary of his finances throughout the year:

January - May → Monthly losses of € 2,475

June - August → Monthly earnings of € 8,230

September → Earnings of € 1,800

October - December → Monthly losses of € 3,170

What was the balance at the end of the year?

Exercise 20. How many years are there from 234 B.C. (before Christ) to 2009 A.D. (after Christ)?

Exercise 21. Aristotle was born in 384 BC and he lived for 64 years. What year did he die?

Answer to exercise 16: a) -4; b) -4; c) -2; d) +4; e) -4; f) +15; g) +8; h) -7;

Answer to exercise 18: He died when he was 33; 2332 years have passed.

Answer to exercise 19: € 12,835



WATCH THESE **VIDEOS** ON THE INTERNET, THEY HELP YOU TO REVISE THIS UNIT ABOUT
“INTEGERS”:
COME ON!



http://www.bbc.co.uk/scotland/learning/bitesize/standard/maths_i/numbers/integers_activity.shtml

<http://www.math-videos-online.com/adding-and-subtracting-integers.html>

<http://www.math-videos-online.com/explanation-on-order-of-operations.html>
