UNIT 2. DIVISIBILITY

1.MULTIPLES AND FACTORS

1.1.Concept of multiple

We say that a number "a" **is a multiple** of another number "b" if the division a : b is an **exact division**, that is, if "b" contains "a" a whole number of times.

(Para obtener los múltiplos de un número lo multiplicamos por 1, 2, 3 y así sucesivamente.)

Solved Example:

Obtain some multiples of 3, 5 and 7:

3 • 1, 3 • 2, 3 • 3, 3 • 4, 3 • 5, 3 • 6 .... so Multiples of 3 = 3, 6, 9, 12, 15, 18, ...

5 • 1, 5 • 2, 5 • 3, 5 • 4, 5 • 5, 5 • 6 .... so Multiples of 5 = 5, 10, 15, 20, 25, 30, ...

7 • 1, 7 • 2, 7 • 3, 7 • 4, 7 • 5, 7 • 6 .... so Multiples of 7 = 7, 14, 21, 28, 35, 42, ...

1.2. Concept of factor

We say that a number "b" **is a factor** of another number "a" if the division a : b is an exact division.

In Spanish:

Por tanto, si la división a : b es exacta, entonces a (el número más grande) es el múltiplo y b (el número más pequeño) es el divisor.

(Para encontrar los divisores de un número debemos hacer probar a dividir por todos los números naturales que son más pequeños que él. Pero hay un pequeño truco que es irlos agrupando por parejas de divisores: Empezamos dividiendo por 1, 2, 3... y si encontramos un divisor el cociente es otro divisor. Seguimos así hasta que empiecen a repetirse).

Solved Example:

Obtain all the factors of 90:

90 : 1 = 90; 90 : 2 = 45; 90 : 3 = 30; 90 : 4 = not possible; 90: 5 = 18; 90 : 6 = 15; 90 : 7 = n. p.; 90 : 8 = n.p.; 90 : 9 = 10; 90 : 10 = 9

(10 and 9 is repeated, so we have finished)

So, the factors of 90 are: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

Solve the following exercises:

1. Find three multiples of 11 that are between 27 and 90.

2. Work out if 556 is a multiple of 4.

3. Find out if 12 is a factor of 144.

4. Which of these numbers is a factor of 91?

a) 3 b) 7 c) 11 d) 13

5. Work out all the factors of the following numbers:

a) 24

b) 27

c) 48

d) 25

e) 7

f) 56

6. Say which of these numbers have exactly three factors.

a) 4

b) 25

c) 15

d) 49

1.3. The properties of multiples and factors

Multiples	Factors
a) Every number is the mutiple of itself Cada número es múltiplo de sí mismo Example: 3 is the multiple of 3	a) Every number is the factor of itself Todo número es divisor de si mismo Example: 3 is the factor of 3
b) Every number is the multiple of 1: Todos los números son múltiplos de 1 Example: 7 is the multiple of 1	b) 1 is the factor of any number 1 es divisor de cualquier número Example: 1 is the factor of 7
c) Zero is the multiple of any number cero es múltiplo de cualquier número Example: 0 is the multiple of 3	c) Zero is not the factor of any number Cero no es divisor de ningún número Example: Zero is not the factor of 2
d) Every number has an infinite number of multiples. Todos los números tienen infinitos múltiplos	d) The set of the factors of a number is finite El conjunto de divisores de un número es finito

2. PRIME AND COMPOSITE NUMBERS

A **prime number** has only two factors: number one and itself. For example: 3, 5, 11, 17, etc.

A **composite number** has more than two factors. For example: 4, 9, 15, 30, etc.

Para averiguar si un número es primo o compuesto puedes hallar sus divisores, o bien dividirlo por todos los números primos menores que él, si no encuentras ningún divisor, entonces el número es primo.

A clever procedure to find the first prime numbers is the Sieve of Erathostenes. It consists of a table with the numbers from 1 to 100, like the one below, and now do the following rules:

- Number 2 is a prime number. Circle it, then cross out all the multiples of 2
- Circle the next number that is not crossed out (3) because it is a prime number too. And then, cross out all its multiples.

- Continue in this way, that is, circle the numbers which are not crossed out and cross out all its multiples until you finish with the table. Then you will have the first prime numbers lower than 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solve the following exercise:

Work out the factors of the numbers below and then point out which ones are prime numbers:

a) 8 b) 101 c) 57 d) 49

3.- DIVISIBILITY RULES

These rules let you test if one number can be evenly divided by another, without having to do too many calculations!

Las reglas de divisibilidad te ayudan a saber si un número es múltiplo de otro sin hacer la división:

● Rule of number 2: A number is divisible by 2 if its last digit is either 0 or an even number. Un número es divisible por 2 si su última cifra es 0 ó un número par. Example: 46, 200, 34, 108.....

● Rule of number 3: A number is divisible by 3 if the sum of its digits is a multiple of 3. Un número es divisible por 3 si la suma de sus cifras es múltiplo de 3. Example: 45, 105, 300, 417....

● Rule of number 4: A number is divisible by 4 if its two last digits are multiples of 4. Un número es divisible por 4 si sus dos últimas cifras son múltiplo de 4. Example: 100, 224, 340, 664....

● Rule of number 5: A number is divisible by 5 if it ends in 0 or 5. Un número es múltiplo de 5 si acaba en 0 ó 5. Example: 200, 345, 650, 800 .....

● Rule of number 9: A number is divisible by 9 if the sum of its digits is a multiple of 9. Un número es divisible por 9 si la suma de sus cifras es múltiplo de 9. Example: 81, 333, 450, 1278.....

● Rule of number 10: A number is divisible by 10 if it ends in 0. Un número es divisible por 10 si acaba en 0. Example: 30, 400, 500.

● Rule of number 11: A number is divisible by 11 if the difference between the sum of the digits on odd positions and the sum of the digits on even positions is 0, 11 or a multiple of 11. Un número es divisible por 11 si la diferencia entre la suma de las cifras en posición par y la suma de las cifras en posición impar es 0, 11 o un múltiplo de 11. Example: 121, 3652

YOU CAN FIND MORE INFORMATION ABOUT RULES OF DIVISIBILITY ON THE INTERNET.
THIS IS A VERY USEFUL AND ENJOYABLE WEBSITE. COME ON! VISIT IT!

http://www.mathsisfun.com/divisibility-rules.html

ALSO, YOU CAN WATCH A VERY INTERESTING VIDEO:

http://www.mathplayground.com/howto_divisibility.html

Solve the following exercises:

1. Use the divisibility rules to complete the following table:

Divisible by	2	3	4	5	9	10	11	25	100
375									
990									
1.848									
12.300									
14.240									

2. Find out two numbers with five digits that are divisible by both 2 and 5 and aren't divisible by 100.

3. Write down two numbers with five digits that are multiples of:

a) 3 and 11 but not of 9.

b) 9 and 11. Are they multiples of 3?

4.- PRIME FACTOR DECOMPOSITION OF A NUMBER

We can write a composite number as a product of several numbers; sometimes we can even write it as several different products:

(Spanish: Cada número compuesto puede escribirse como un producto de números, a veces incluso como varios productos distintos):

Example: $15 = 5 \cdot 3$

$24 = 2 \cdot 12 = 3 \cdot 8 = 3 \cdot 2 \cdot 4 = 24 \cdot 1 = \ldots.$

But we can write every number as a product of prime numbers which is unique. Finding that product of prime numbers is what we call **prime factor decomposition of a number**.

(Spanish: Pero cada número puede ser escrito únicamente como un producto de números primos que es único. Encontrar ese producto es lo que llamamos descomposición en factores primos.)

If we hava a small number we can do the prime factor decomposition in our heads, but remember you can only use prime numbers:

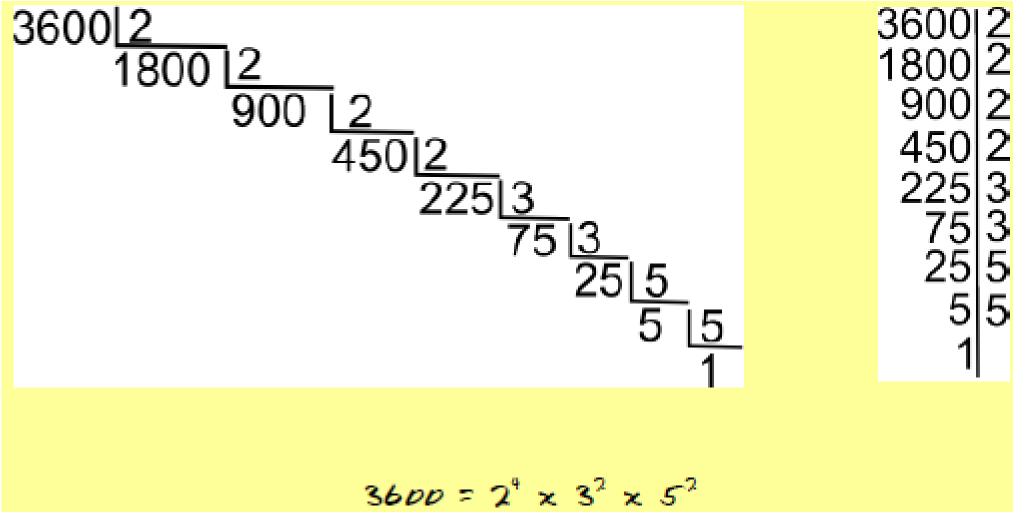
(Spanish: Si tenemos un número pequeño podemos hacer la descomposición mentalmente, pero recuerda sólo puedes usar números primos:)

Examples: 6 = 2 • 3

24 = 4 • 6 = 2 • 2 • 2 • 3 = 2^3 • 3

Solved Example:

Work out the prime decomposition of 3600:



Tip: If the number ends in zero, you can change each zero by the factors 2 • 5, so if the last two digits are zeros, the prime decomposition will have 2^2 • 5^2.

(Spanish: Truco: Cuando el número acabe en 0, se puede cambiar cada cero por los factores 2 x 5, así que si las dos últimas cifras son cero la descomposición en factores primos tendrá 2^2 • 5^2).

Solve the following exercises:

1. Work out the prime factor decomposition of the following numbers:

a) 108 b) 99 c) 42

d) 37 e) 100 f) 840

2. Complete these prime factor decompositions:

a) $360 = 2^{?} \times ?^{?} \times 5$ b) $300 = ?^{?} \times ? \times 5^{2}$

5. THE HIGHEST COMMON FACTOR AND THE LEAST COMMON MULTIPLE

5.1. Concept of the highest common factor (HCF)

To understand the concept, check this example to calculate HCF (30, 48, 54).

Firstly, calculate:

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Now, we choose the common factors: **(Ahora vamos a elegir los divisores comunes a los tres números)**

Common Factors of 30, 48 and 54 : 1, 2, 3, 6

Which is the biggest one? It is 6, so:

HCF (30, 48, 54) = 6

Definition:

The highest common factor of several numbers is the largest number that evenly divides into all of them.

Spanish:____

____

5.2. Rule for calculating the H.C.F.

A veces puede llevar mucho tiempo averiguar todos los divisores de varios nombres, por lo que hace falta un método más sencillo.

Rule:

"To work out the HCF of several numbers, first you have to find the prime factor decomposition of the given numbers and then, to take the common factors with the least index".

Spanish: ____

____

Solved example:

Find out the HCF of numbers 36, 48 y 90.

1^{st} step: Write them as a product of prime factors:

$36 = 2^2 \cdot 3^2$ $48 = 2^4 \cdot 3$ $90 = 2 \cdot 3^2 \cdot 5$

2^{nd} step: Take the common factors with the least index:

H.C.F. = 2 • 3 = 6

We can also do it in the English way. It consists of writing all the factors of each number in a row and then mark the common ones.

36 = 2 • 2 • 3 • 3

48 = 2 • 2 • 2 • 2 • 3

90 = 2 • 3 • 3 • 5

Señalamos los factores que sean comunes en los tres números:

36 = **2** • 2 • **3** • 3

48 = **2** • 2 • 2 • 2 • **3** $\Rightarrow$ H.C.F. = 2 • 3 = 6

90 = **2** • 3 • **3** • 5

Solve the following exercises:

1. Work out the factors of the numbers below and then find out the HCF:

a) 2 and 16

b) 3 and 25

c) 9, 12 and 18

d) 27, 36 and 63

2. Find out the HCF of the following numbers using the Spanish and the English methods:

a) 4, 6, 18 and 32

b) 3, 4, 12, 36 and 48

5.3. Concept of the least common multiple (lcm)

In this case, we calculate l.c.m. (2,3):

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,...

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36,...

Now, we choose the common multiples:

Multiples of 2 and 3: 6, 12, 18, 24, 30, 36,...

Which is the smallest? It is 6, so:

l.c.m. (2,3)= 6

Definition: The least (lowest) common multiple of several numbers is the smallest number that is multiple of all of them.

And in Spanish: ____

____

5.4. Rule for calculating the l.c.m.

Rule:

"To work out the lcm of several numbers, first write them as a product of their prime factors and then take the common and non-common factors with the highest index."

And in Spanish: ____

____

Solved example:

Find out the lcm of numbers 36, 48 and 90:

1^{st} step: First obtain the prime factor decomposition:

$36 = 2^2 \cdot 3^2$ $48 = 2^4 \cdot 3$ $90 = 2 \cdot 3^2 \cdot 5$

2^{nd} step: Now, take the common and non-common factors with the highest index:

$l.c.m = 2^4 \cdot 3^2 \cdot 5 = 720$

A pesar de tener estas reglas es una buena idea acostumbrarse a calcular el m.c.d. y m.c.m. mentalmente cuando los números son pequeños. Sólo tienes que pensar en un múltiplo pequeño o en un divisor grande de los números dados.

Solved example:

Find out mentally the HCF and the lcm of the numbers below:

a) 3 and 5; HCF = 1 and lcm = 15

b) 2 and 4; HCF = 2 and lcm = 4

c) 6 and 15; HCF = 3 and lcm = 30

Solve the following exercises:

1. Work out the l.c.m. of the numbers below:

a) 9, 12 and 18 b) 27, 36 and 63

2. Work out the l.c.m. of the following numbers. What conclusion do you reach?

a) 2, 4, 8 and 16 b) 3, 4, 6 and 12

EXERCISES

UNIT 2. DIVISIBILITY.

(See the exercise solutions at the end of these notes).

Exercise 1. Write:

a) The first five multiples of 11

b) The multiples of 20 between 150 and 210

c) A multiple of 13 between 190 and 200

Exercise 2. Write

a) All the pairs of numbers whose product is 80.

b) All the divisors of 80.

Exercise 3. Find all the divisors of

a) 30 b) 39 c) 45 d) 50

Exercise 4. Find, in each case, all the possible values of "a" so that the result is at the same time a multiple of 2 and of 3:

a) 4a b) 32a c) 24a

Exercise 5. Find the prime numbers greater than 25 and less than 45.

Exercise 6. Sort the prime numbers from the composite numbers:

14 17 28 29 47 53 57 63 71 79 91 99

Exercise 7. Select at a single glance,

$a = 2^5 \cdot 3$ $b = 2^2 \cdot 7^2$ $c = 2 \cdot 3^2 \cdot 5$ $d = 2^2 \cdot 5 \cdot 11$ $e = 3 \cdot 5^2 \cdot 13$ $f = 2^2 \cdot 3^2 \cdot 7$

a) The multiples of 10
b) The multiples of 14
c) The multiples of 15
d) The multiples of 18
e) One that is a multiple of 13
f) One that is a multiple of 30

Exercise 8. Select at a glance,

$a = 2 \cdot 3$ $b = 2 \cdot 5$ $c = 3 \cdot 5$ $d = 2^2 \cdot 3$ $e = 2^2 \cdot 5$ $f = 2 \cdot 5^2$

a) The factors of $20 = 2^2 \cdot 5$
b) The factors of $30 = 2 \cdot 3 \cdot 5$
c) The factors of $60 = 2^2 \cdot 3 \cdot 5$
d) The factors of $90 = 2 \cdot 3^2 \cdot 5$

Exercise 9. Work out the lowest common multiple of a and b in each case:

a) $a = 48$; $b = 56$ b) $a = 80$; $b = 88$ c) $a = 175$; $b = 350$

Exercise 10. Calculate the greatest common factor of *a* and *b* in each case:
a) $a = 63$; $b = 84$ b) $a = 105$; $b = 120$ c) $a = 165$; $b = 198$

Exercise 11. Work out:
a) lcm(2, 4, 8) b) GCF(2, 4, 8) c) lcm(10, 15, 20) d) GCF(10, 15, 20)
e) lcm(20, 30, 40) f) GCF(20, 30, 40)

Exercise 12. Find out the values of *a* and *b* knowing that their lcm(a, b) = 20 and their HCF(a, b) = 2

Exercise 13. Find all the possible forms of making piles of equal size with 72 sugar cubes.

Exercise 14. Ricardo can arrange his collection of picture cards into pairs, trios and groups of five. How many cards does Ricardo own knowing that there are more than 80 and less than 100?

Exercise 15. A glass weighs 75 gr. and a cup weighs 60 gr. How many glasses do you have to put on a balance plate and how many cups on the other so that they are in balance?

Exercise 16. A market vender exchanges with a mate a batch of t-shirts for 24 € per piece for a batch of trainers for 30 € per piece. How many t-shirts does he give and how many pairs of trainers does he receive?

Exercise 17. In a timber yard there is a stack of pine planks. Each plank is 35 mm. thick, and the stack is the same height as a stack of oak planks which are 90 mm. thick. What is the height of both stacks? (There are at least three solutions)

Exercise 18. A group of 60 kids are accompanied by 36 parents to go to a camp in the mountains. They agree to have the same number of people in each cabin. The fewer cabins they occupy the less they pay. On the other hand, the parents don't want to sleep with the kids nor do the kids want to sleep with their parents. How many people will sleep in each cabin?

Answers:
Exercise 1: a) 11, 22, 33, 44 y 55; b) 160, 180 y 200; c) 195;
Exercise 2: a) 1 · 80 = 2 · 40 = 4 · 20 = 5 · 16 = 8 · 10; b) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80;
Exercise 3: a) 1, 2, 3, 5, 6, 10, 15, 30; b) 1, 3, 13, 39; c) 1, 3, 5, 9, 15, 45; d) 1, 2, 5, 10, 25, 50
Exercise 4: a) 2 y 8; b) 4; c) 0 y 6;
Exercise 5: 29, 31, 37, 41 y 43
Exercise 6: prime numbers: 17, 29, 47, 53, 71 y 79; composite numbers: 14, 28, 57, 63, 91 y 99
Exercise 7: a) c y d; b) b y f; c) c y e; d) c y f; e) e; f) c;
Exercise 8: a) b y e; b) a, b y c; c) a, b, c, d y e; d) a, b y c;
Exercise 9: a) 336; b) 880; c) 350;
Exercise 10: a) 21; b) 15; c) 33;
Exercise 11: a) 8; b) 2; c) 60; d) 5; e) 120; f) 10;
Exercise 12: (10 and 4) or (20 and 2)
Exercise 13: 72 of 1; 36 of 2; 24 of 3; 18 of 4; 12 of 6; 9 of 8; 8 of 9: 6 of 12; 4 of 18; 3 of 24; 2 of 36; 1 of 72
Exercise 14: Ricardo owns 90 picture cards
Exercise 15: 4 glasses and 5 cups;
Exercise 16: 5 t-shirt for 4 pairs of trainers
Exercise 17: it must be a multiple of 14 cm.
Exercise 18: 12 people in each cabin

TO PRACTISE MORE EXERCISES ABOUT DIVISIBILITY YOU CAN DO IT ON THE INTERNET, THIS IS A VERY USEFUL AND ENJOYABLE WEBSITE. COME ON! TRY IT!

http://webs.ono.com/paco_garces/1ESO/index.html

GAMES

Have you finished? Then you can play these games!

http://www.jamit.com.au/htmlFolder/app1004.html

http://www.mathplayground.com/calculator_chaos.html

I hope you have learnt a lot!